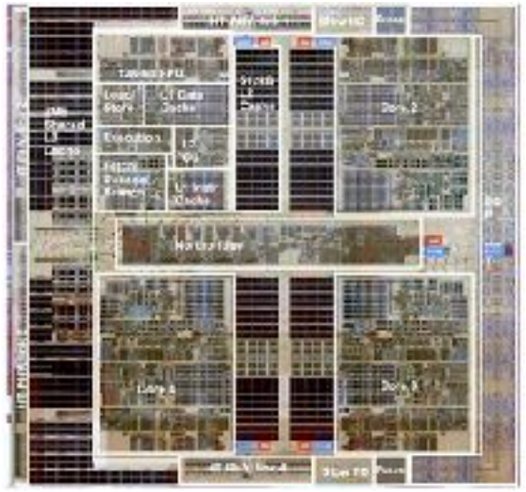
Shared-Resource-Centric Limited Preemptive Scheduling: A Comprehensive Study of Suspension-based Partitioning Approaches

Zheng Dong Cong Liu Soroush Bateni
The University of Texas at Dallas, USA

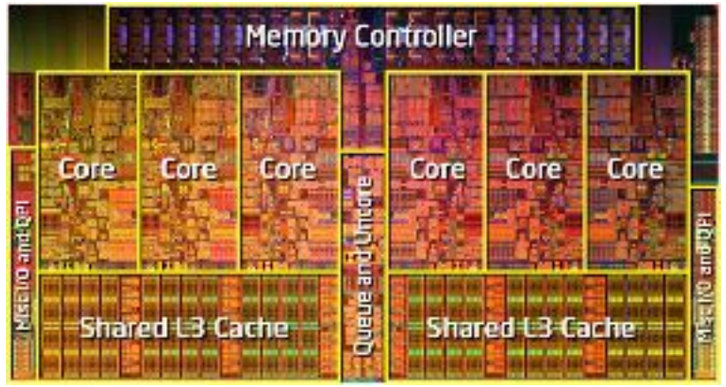
Kuan-Hsun Chen Jian-Jia Chen Georg von der Bruggen Junjie Shi
Technical University of Dortmund, Germany

Multi-Core Processor

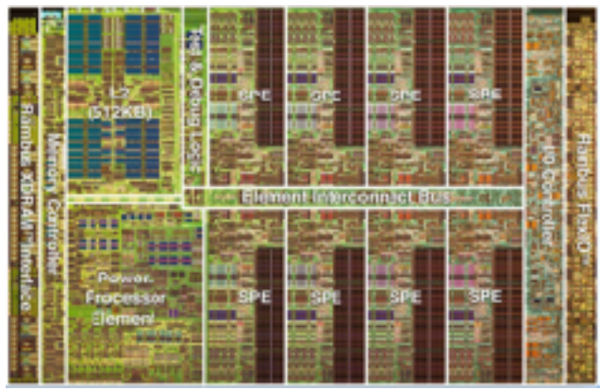
- Simpler and lower power than a single large core
- Smaller inter-core communication latency



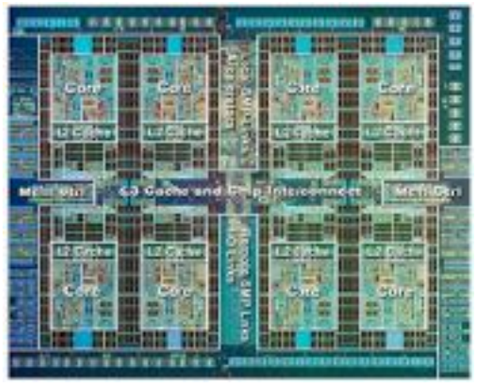
AMD Barcelona (4 cores)



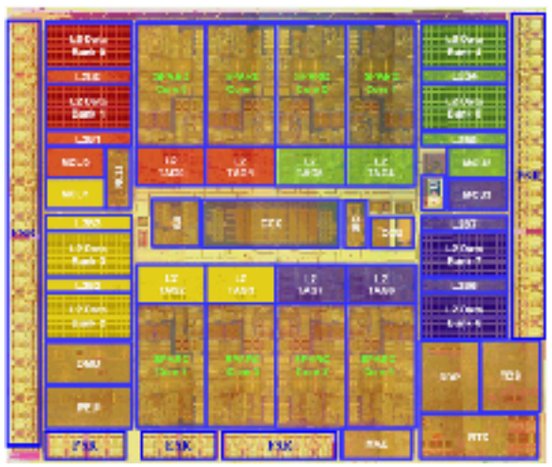
Intel Core i7 (8 cores)



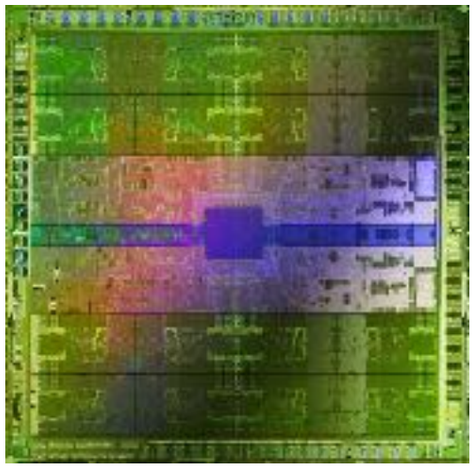
IBM Cell BE (8+1 cores)



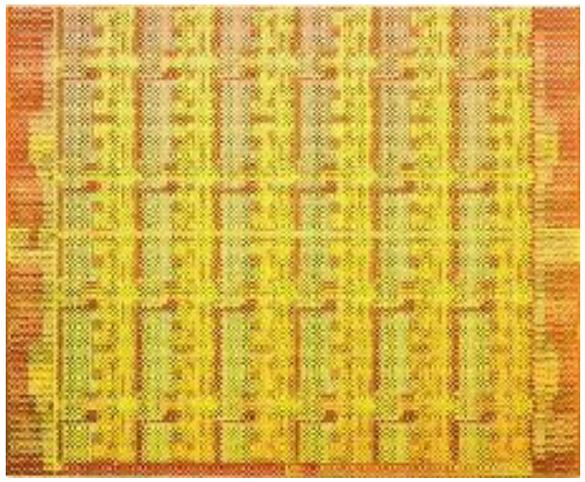
IBM POWER7 (8 cores)



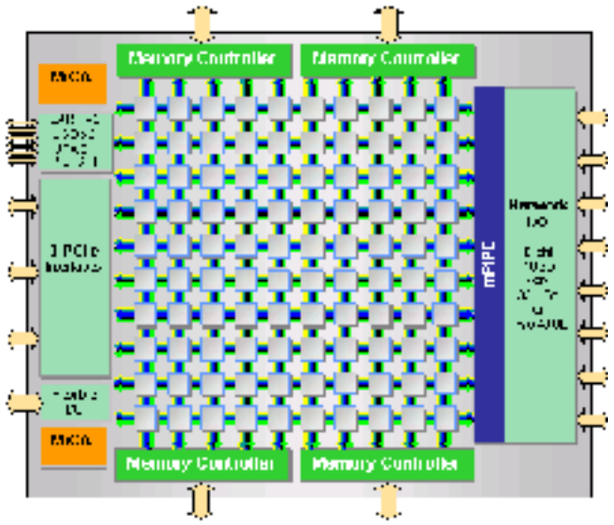
Sun Niagara 2 (8 cores)



Nvidia Fermi (448 cores)



Intel SCC (48 cores)



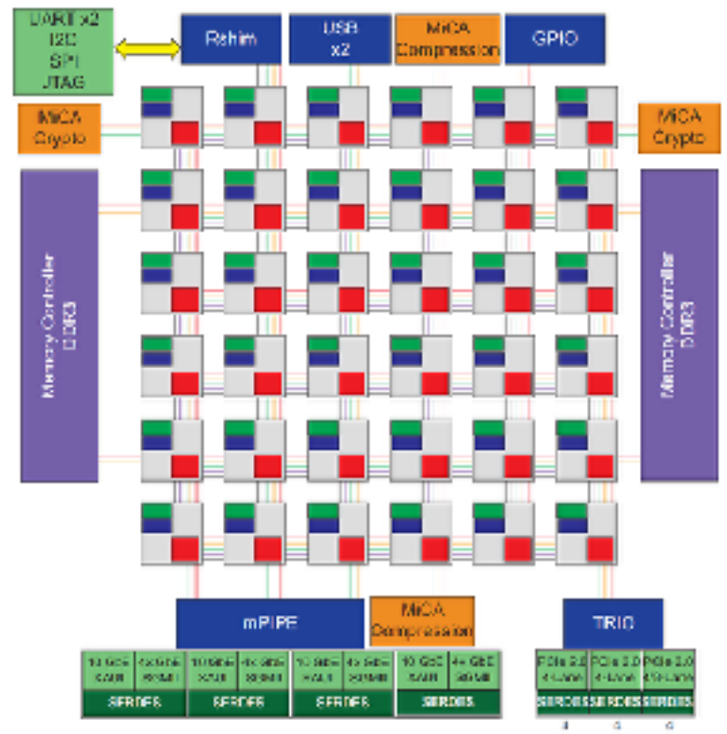
Tilera TILE Gx (100 cores)

An example of multi-phase task

Autonomous Driving Vehicle

An example of multi-phase task

Autonomous Driving Vehicle



100 cores on chip

An example of multi-phase task

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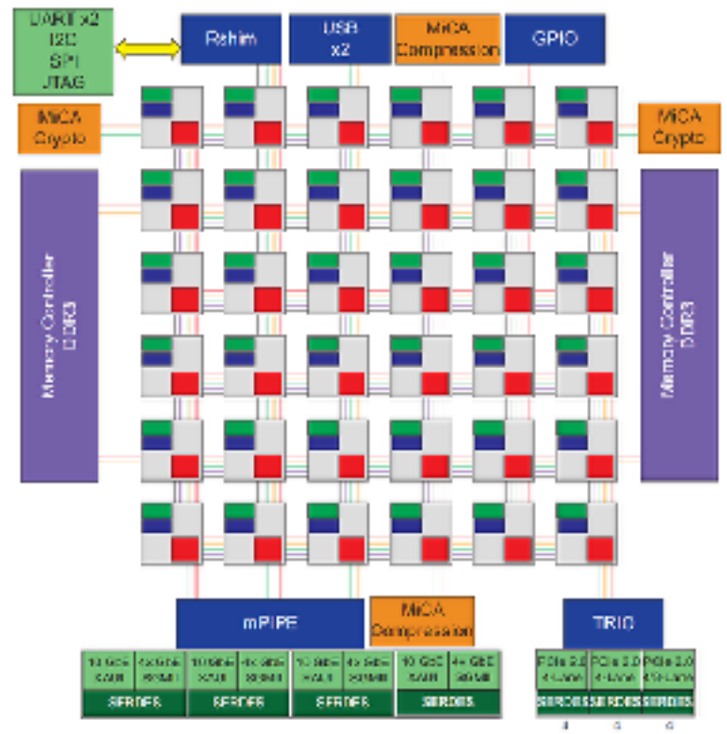

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GPU (shared resource)

An example of multi-phase task

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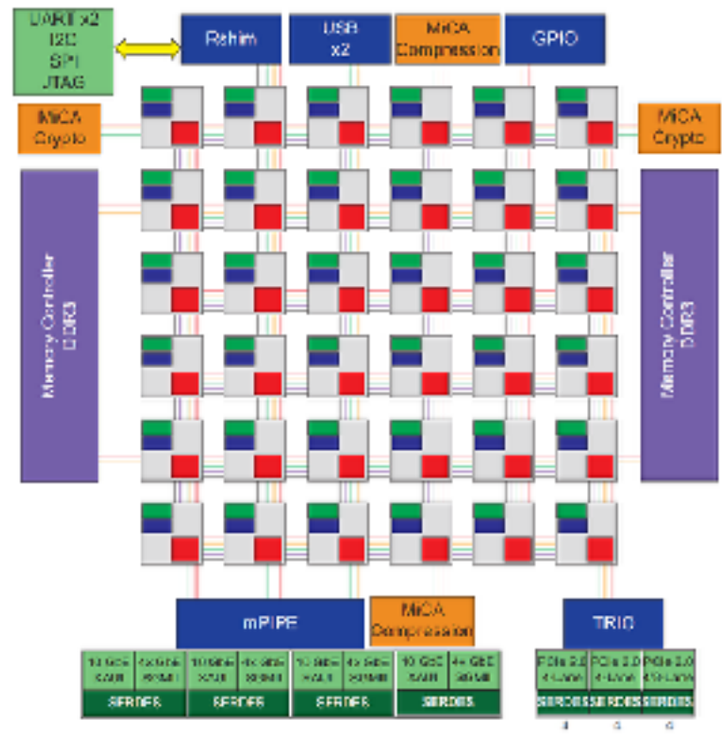

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- CPUs and GPU **communicate** with each other constantly through the PCI **bus.**

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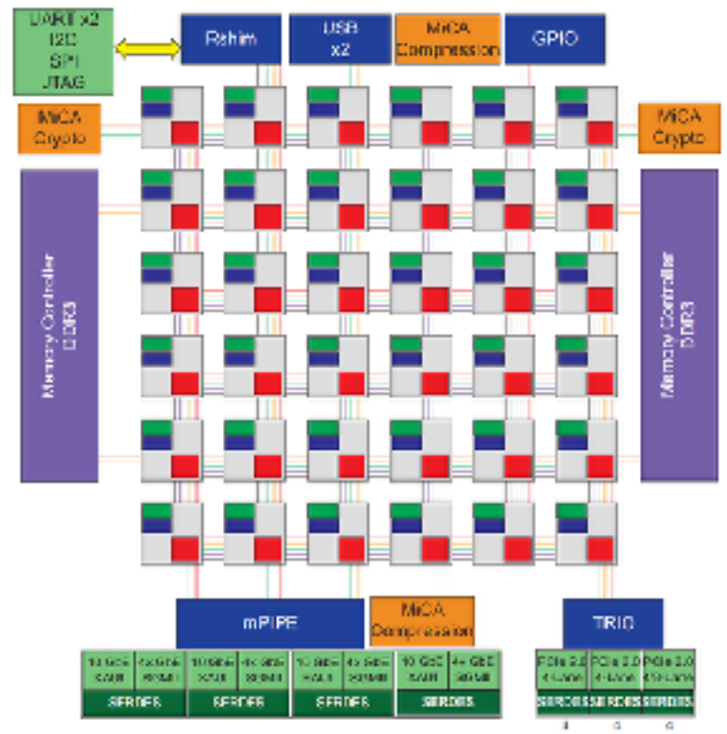

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Multi-phase task:

CPU-phase	GPU-phase	CPU-phase	GPU-phase	CPU-phase

Traditional methods encounter multi-core system

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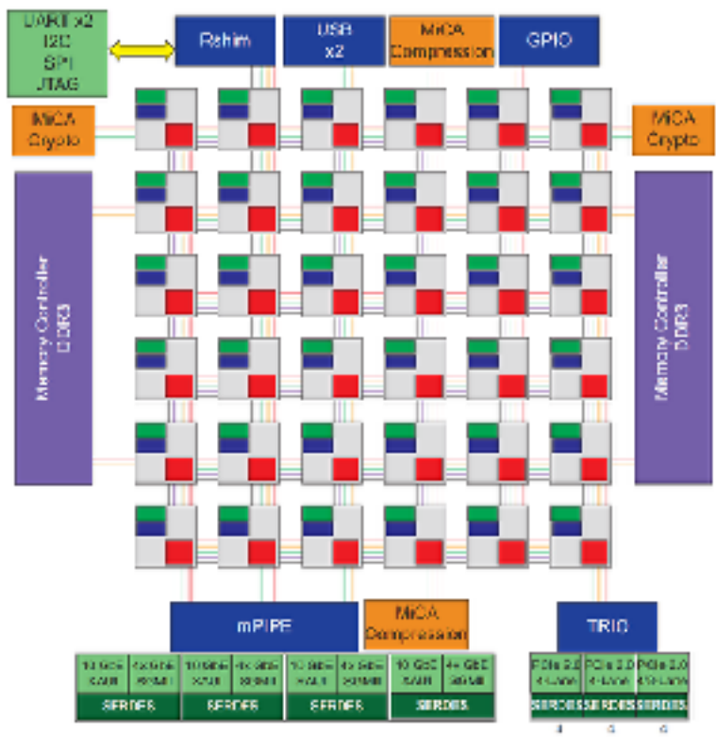

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GPU (accelerator)

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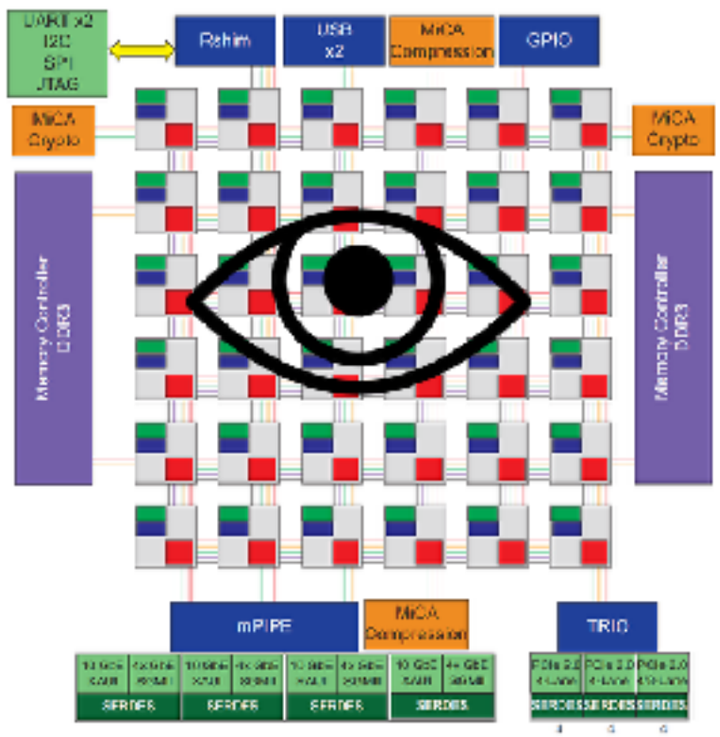

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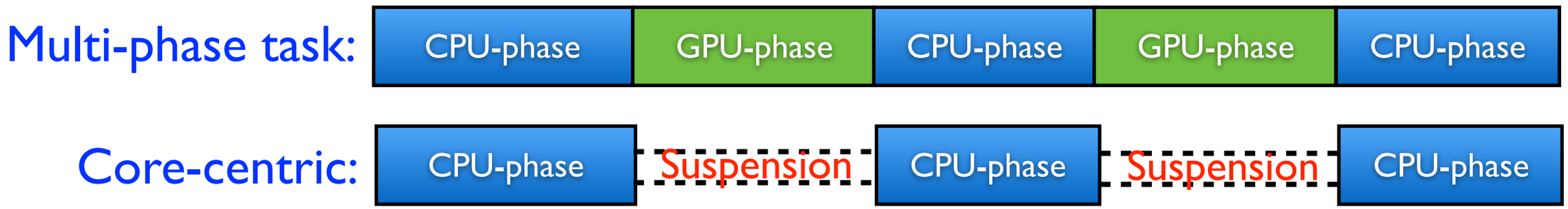

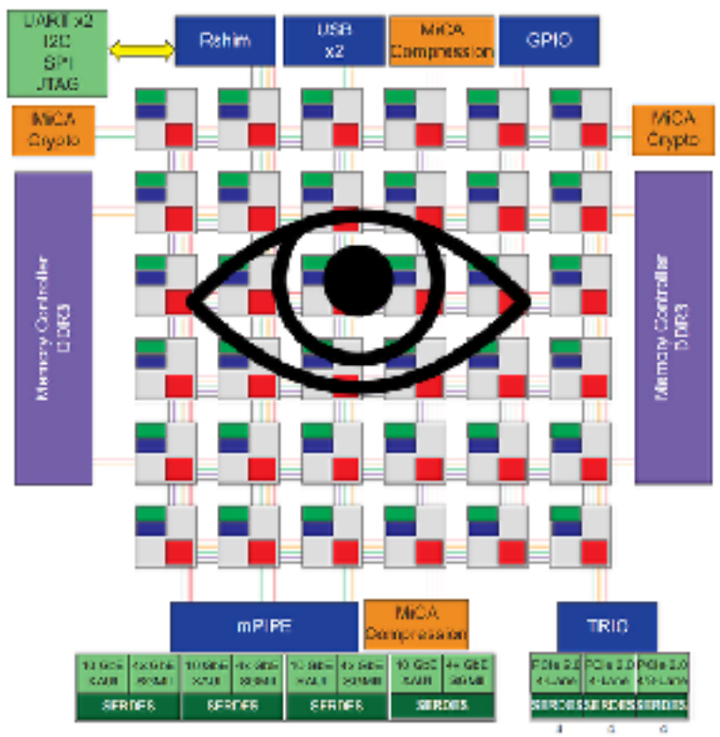

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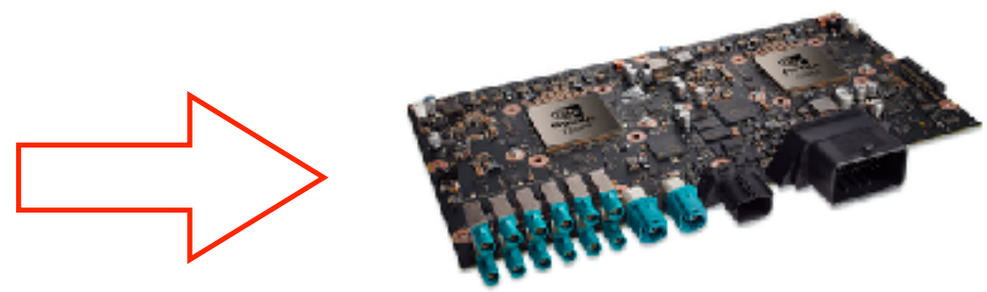

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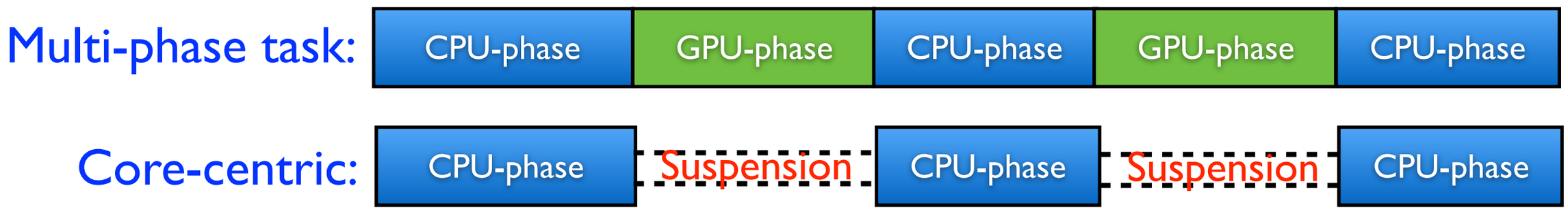

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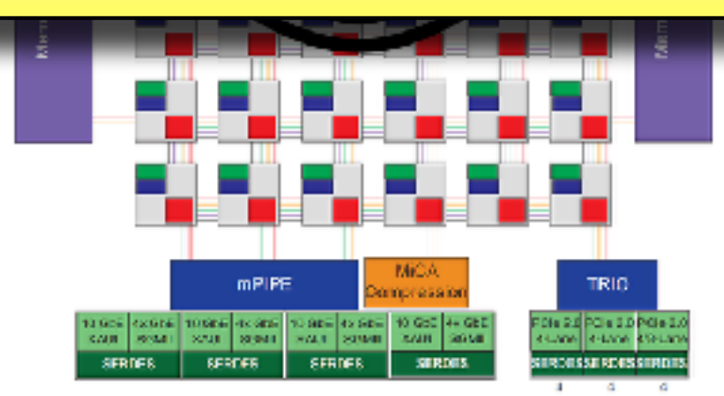

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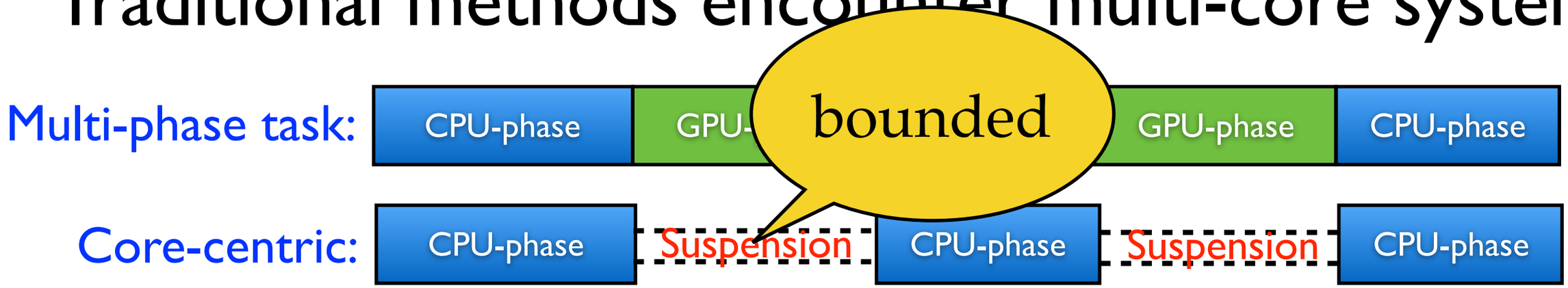

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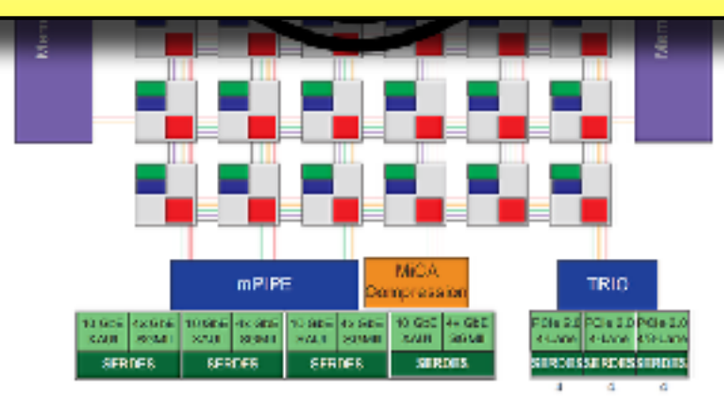

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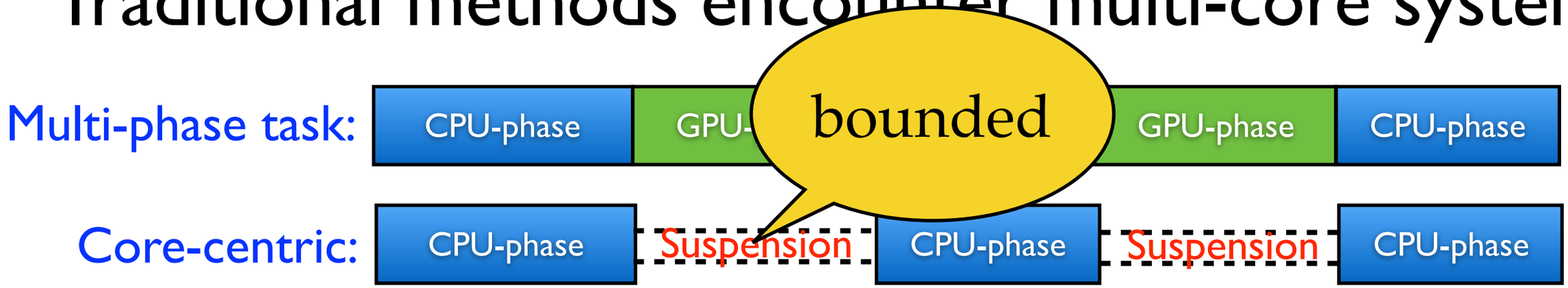

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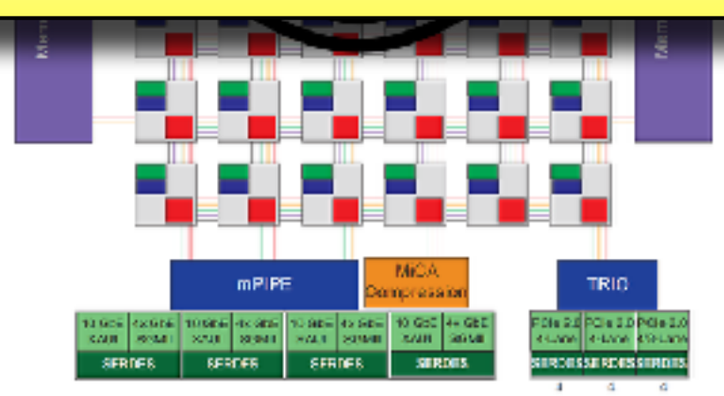

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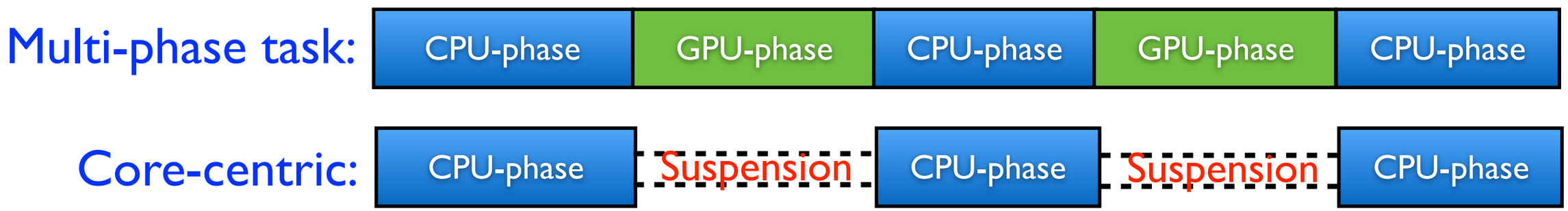
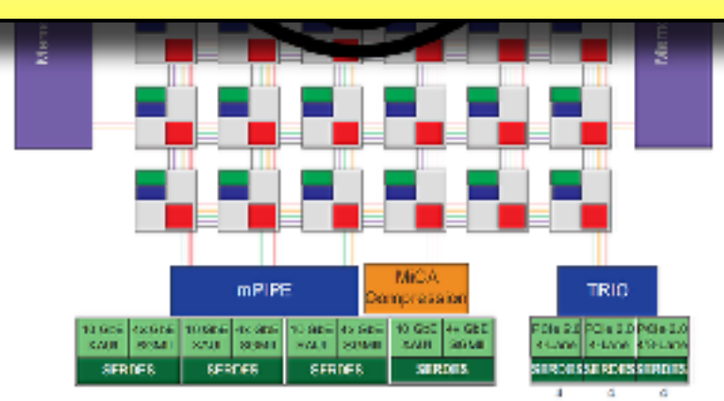


Shared resource may become the actual scheduling bottleneck, causing the worst-case latency bound on shared-resource rather pessimistic or impossible to derive.



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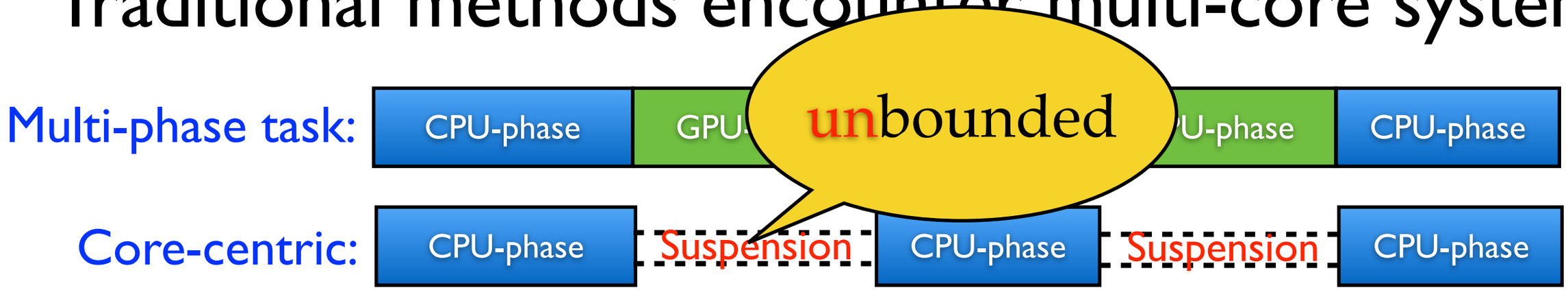

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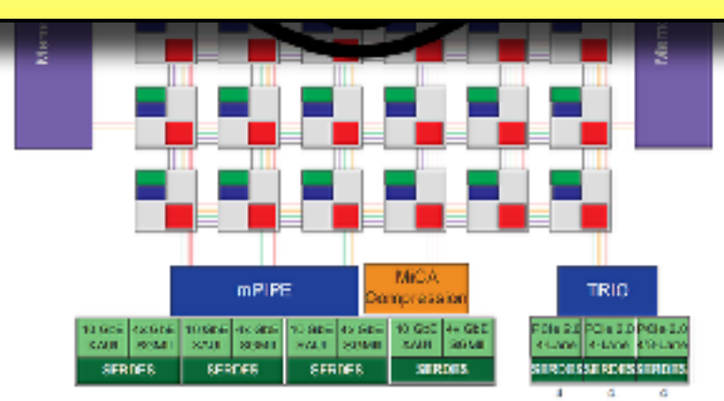

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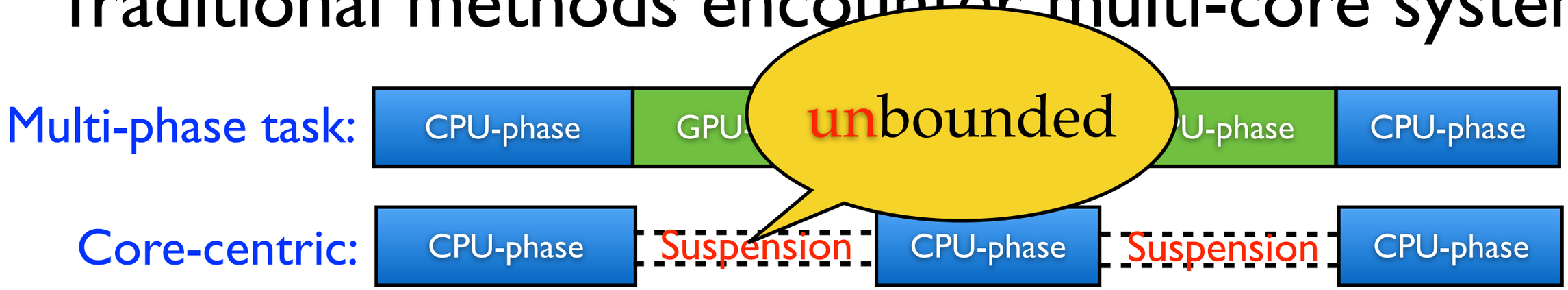

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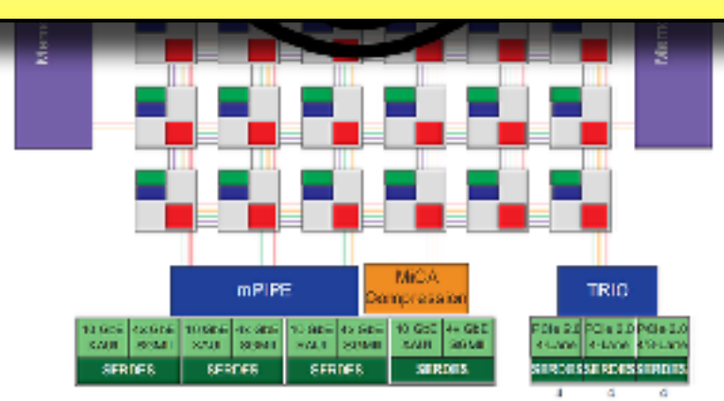

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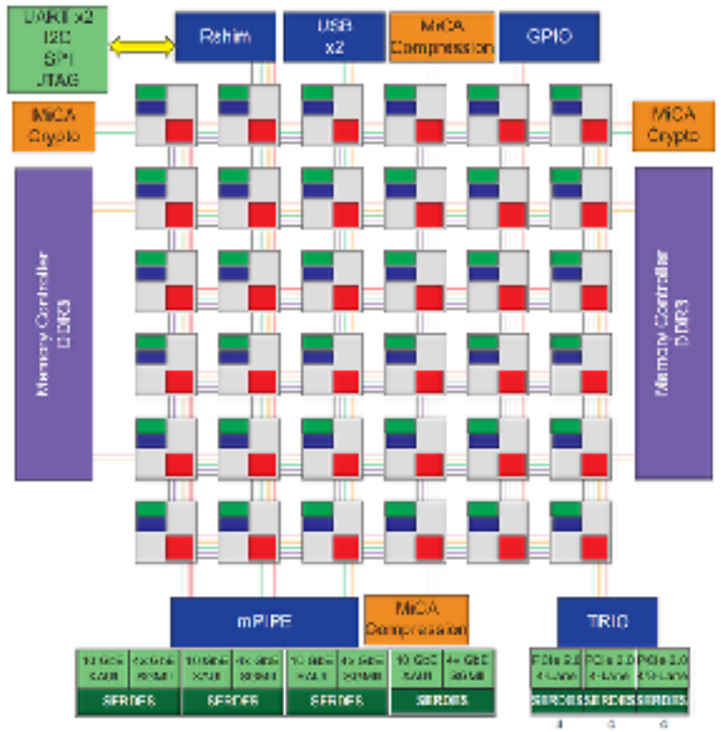
Not Applicable

A counter-intuitive shared-resource-centric perspective

Multi-phase task:

CPU-phase	GPU-phase	CPU-phase	GPU-phase	CPU-phase



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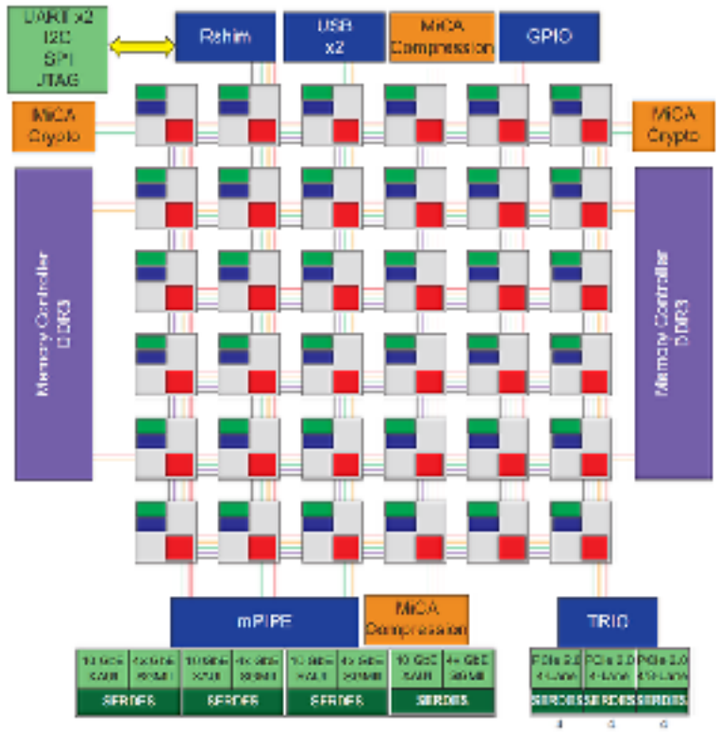

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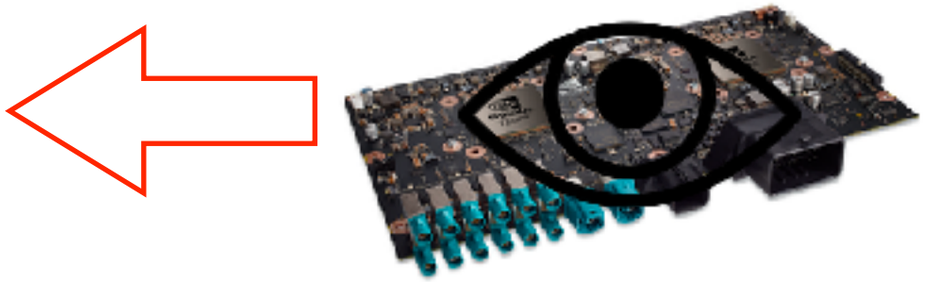

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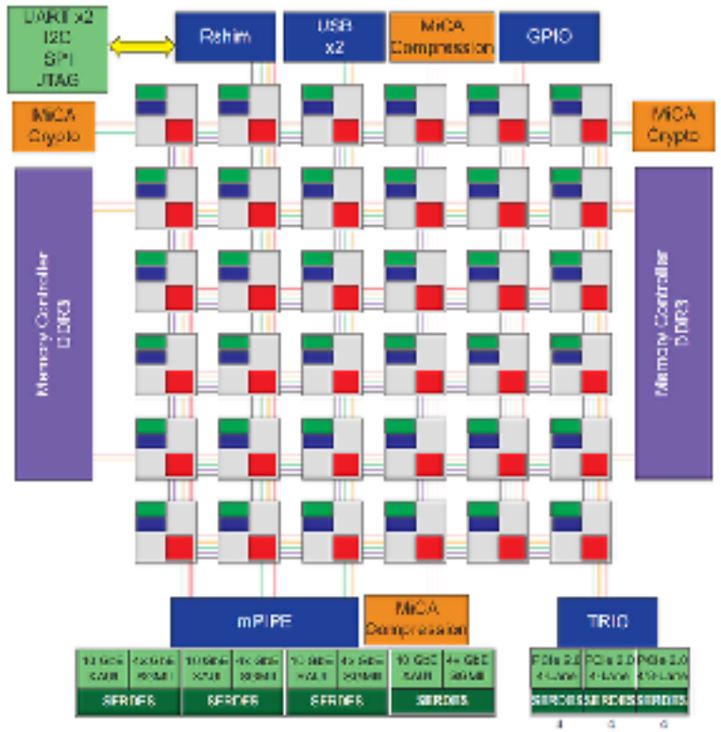
A counter-intuitive shared-resource-centric perspective

Multi-phase task: CPU-phase | GPU-phase | CPU-phase | GPU-phase | CPU-phase

Shared-resource-centric: Suspension | GPU-phase | Suspension | GPU-phase | Suspension



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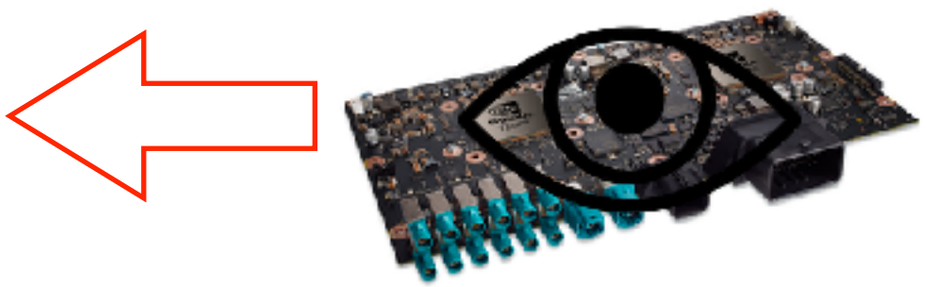

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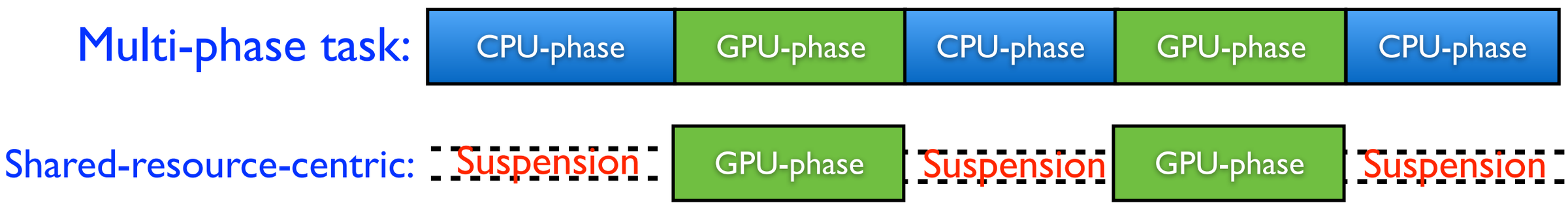

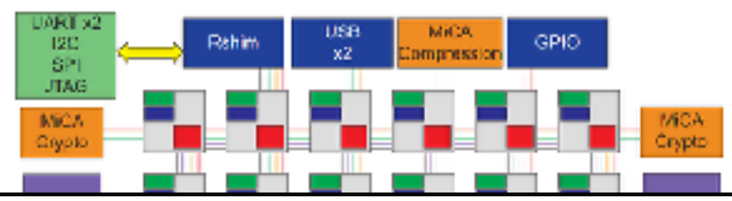

The idea is to judiciously scheduling shared-resource requests (treating shared-resource as the first class unit) and bounding the worst case latency a task may experience on the computing cores (treating cores as "I/O").

100 cores on chip

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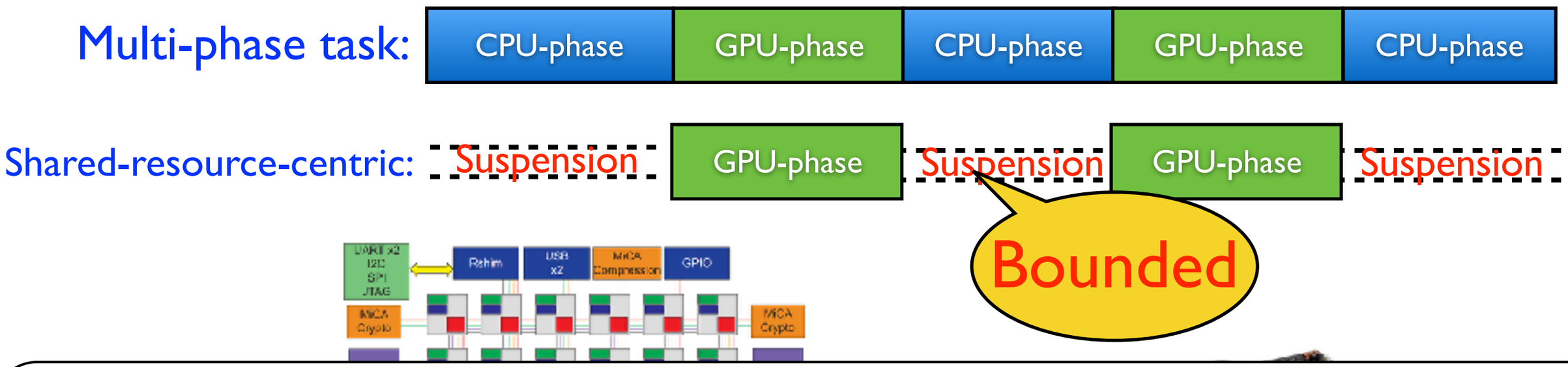

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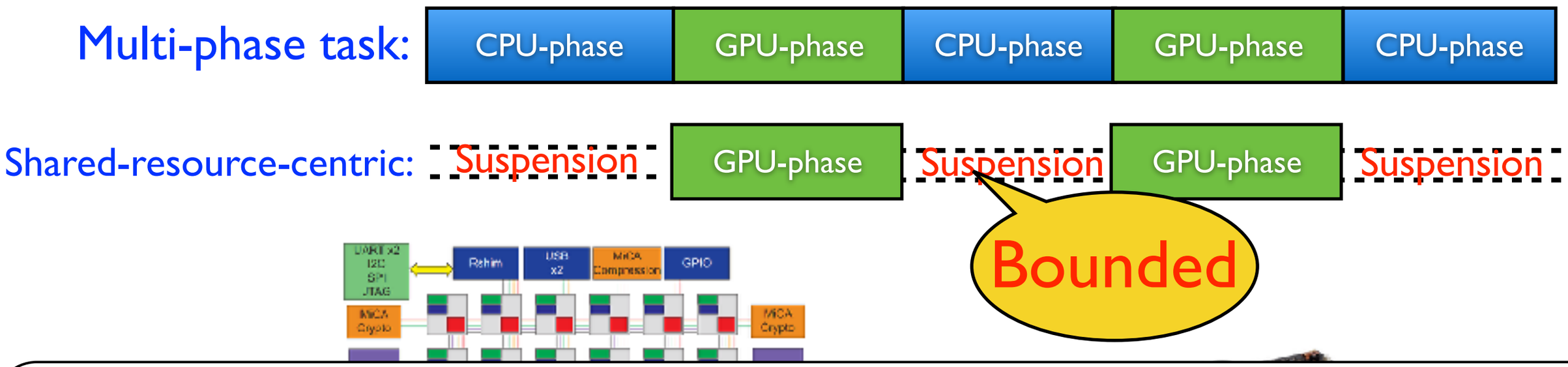

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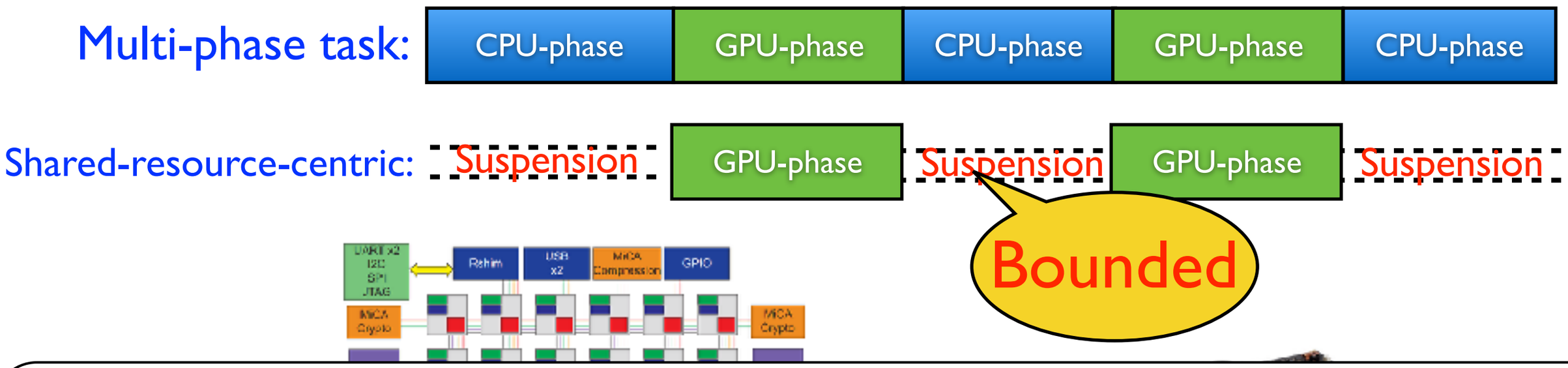

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Challenges in shared-resource-centric scheduling

- shared-resource-centric suspending task model VS. Traditional suspending task model

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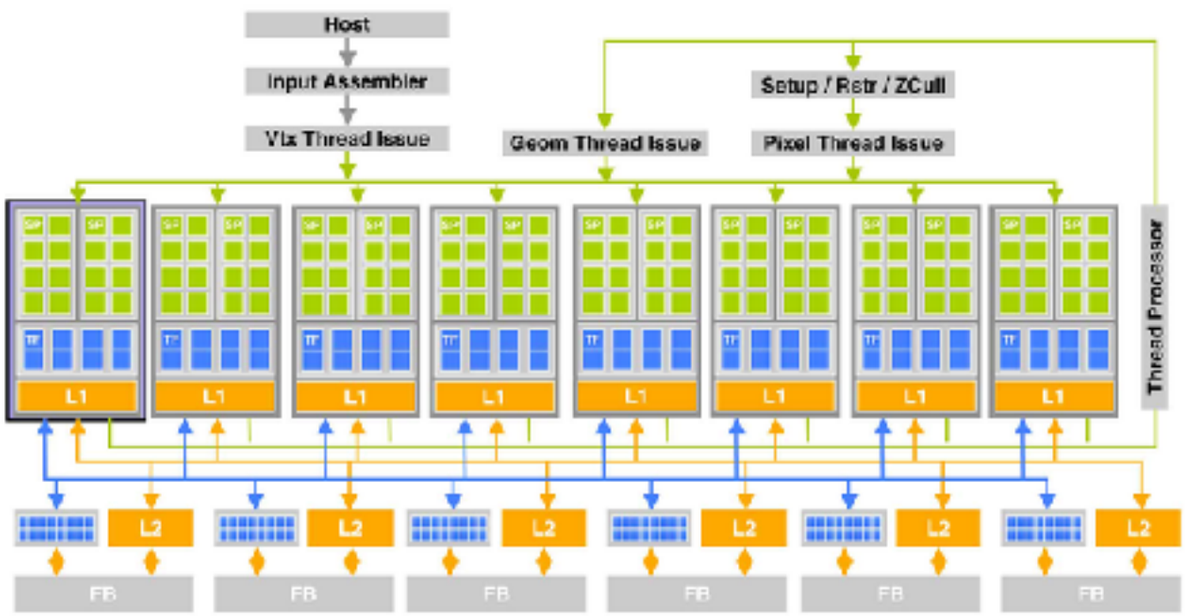
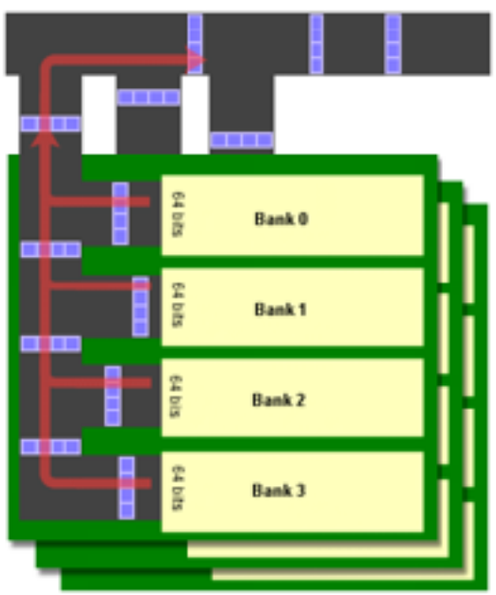


GPU with parallel programmable units at their cores.



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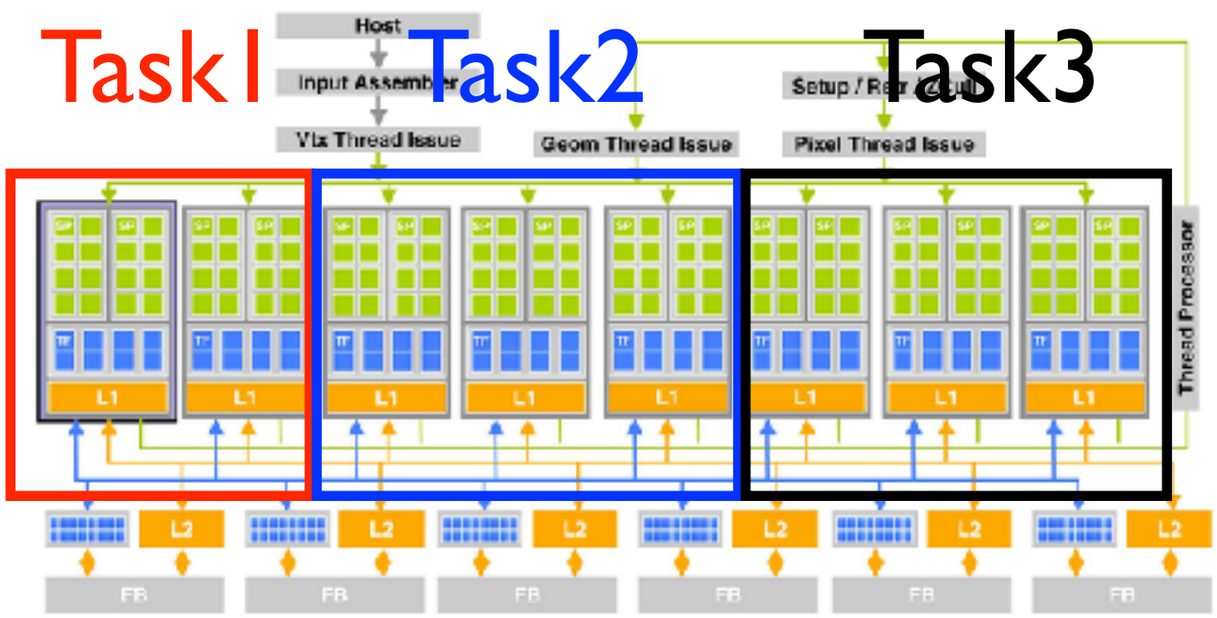

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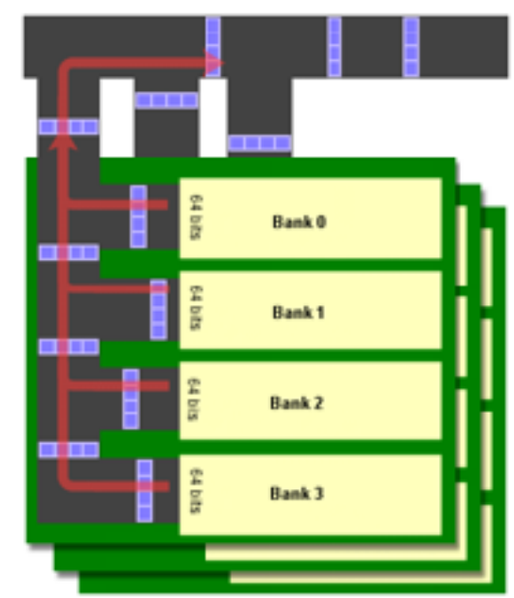

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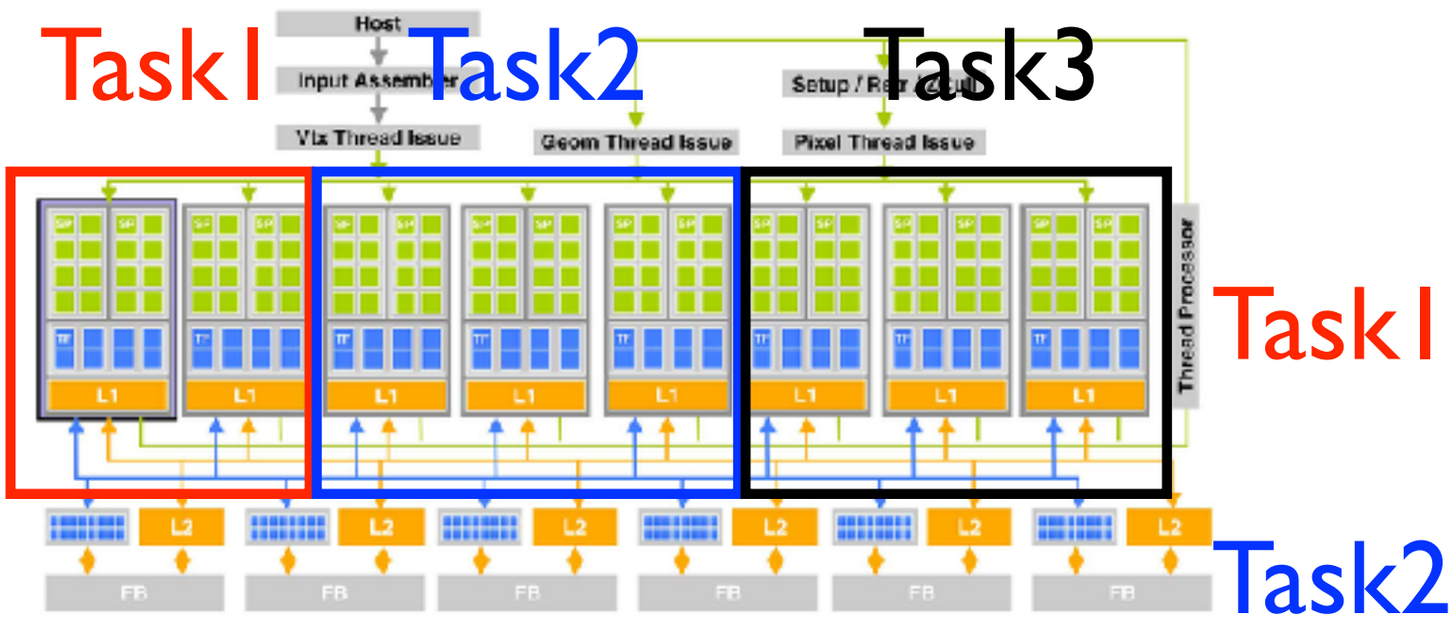

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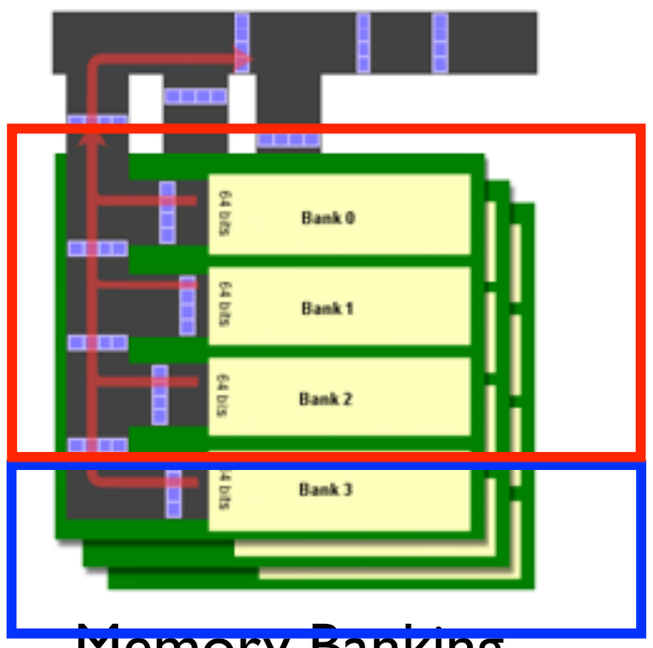

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Objectives and solution overview

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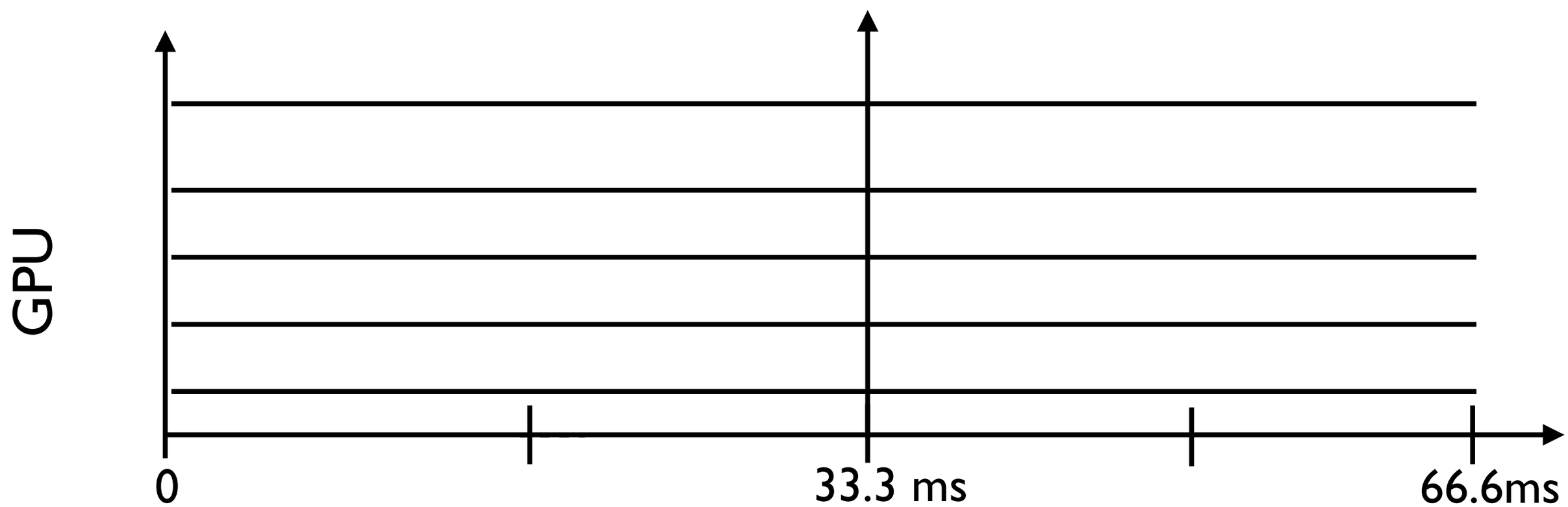
- Solution overview (two steps):
  - Step1: Developing schedulability tests that can guarantee HRT constraints
  - Step2: Partitioning tasks onto a multi-unit shared resource to minimize the amount of required shared resource.

Shared-resource-centric suspending task model

- Implicit-deadline sporadic multi-phase task: τi(ei, si, pi, di, σ_i, zi)



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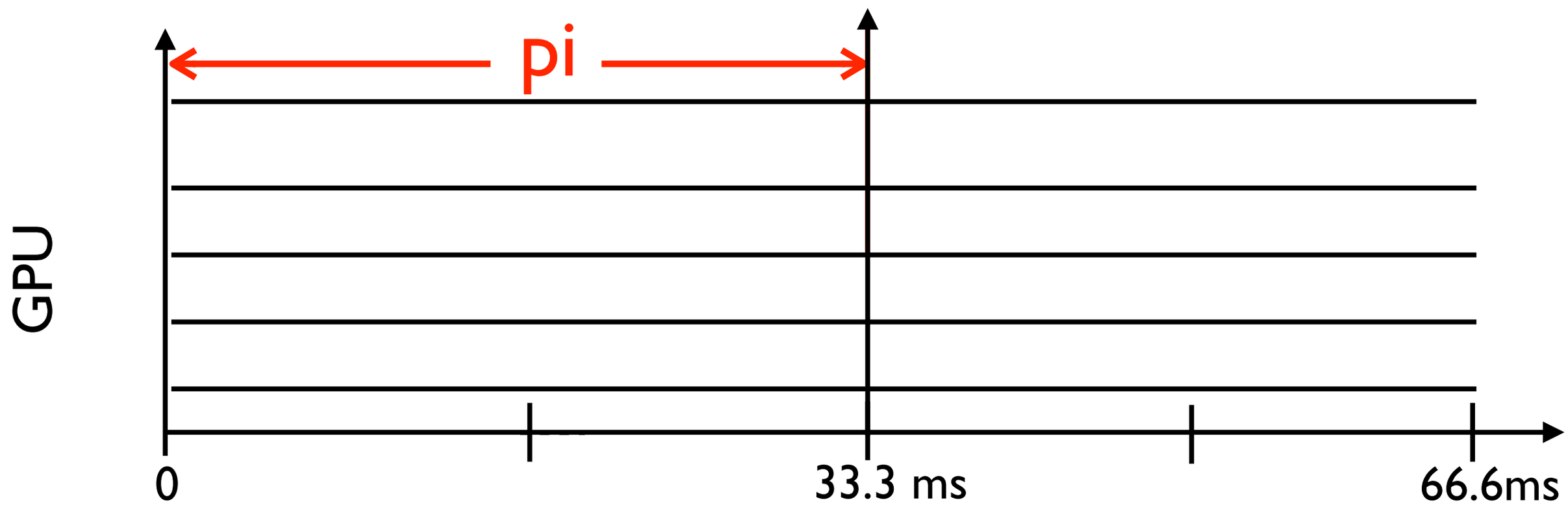

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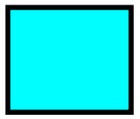
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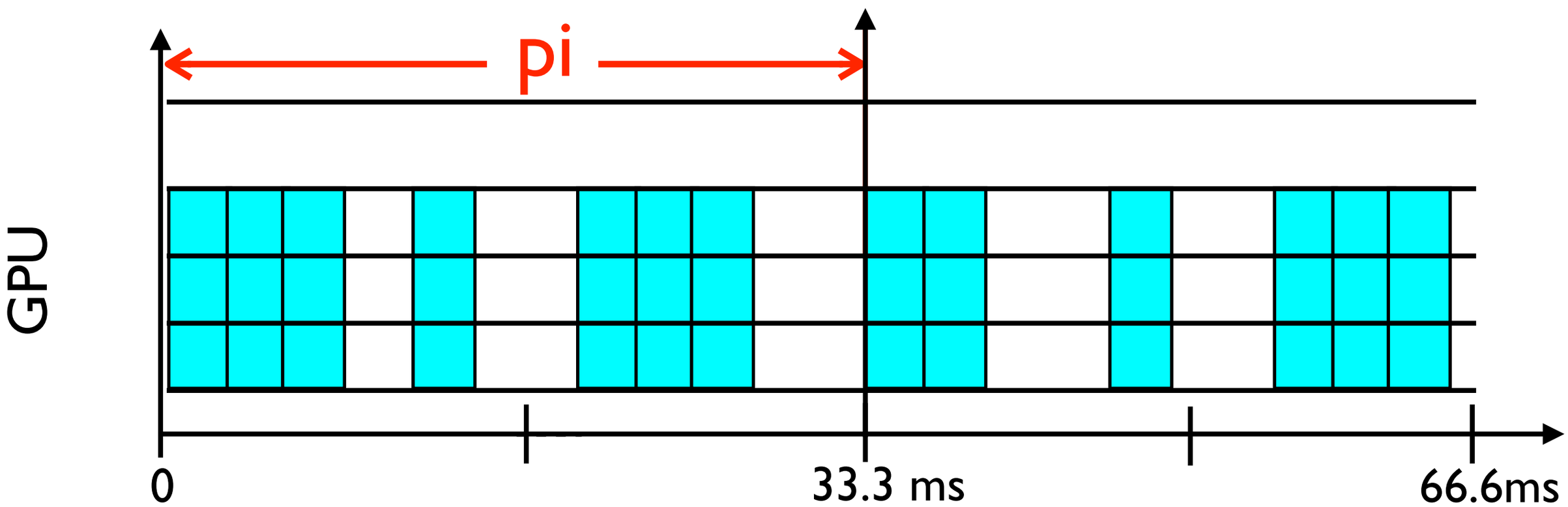

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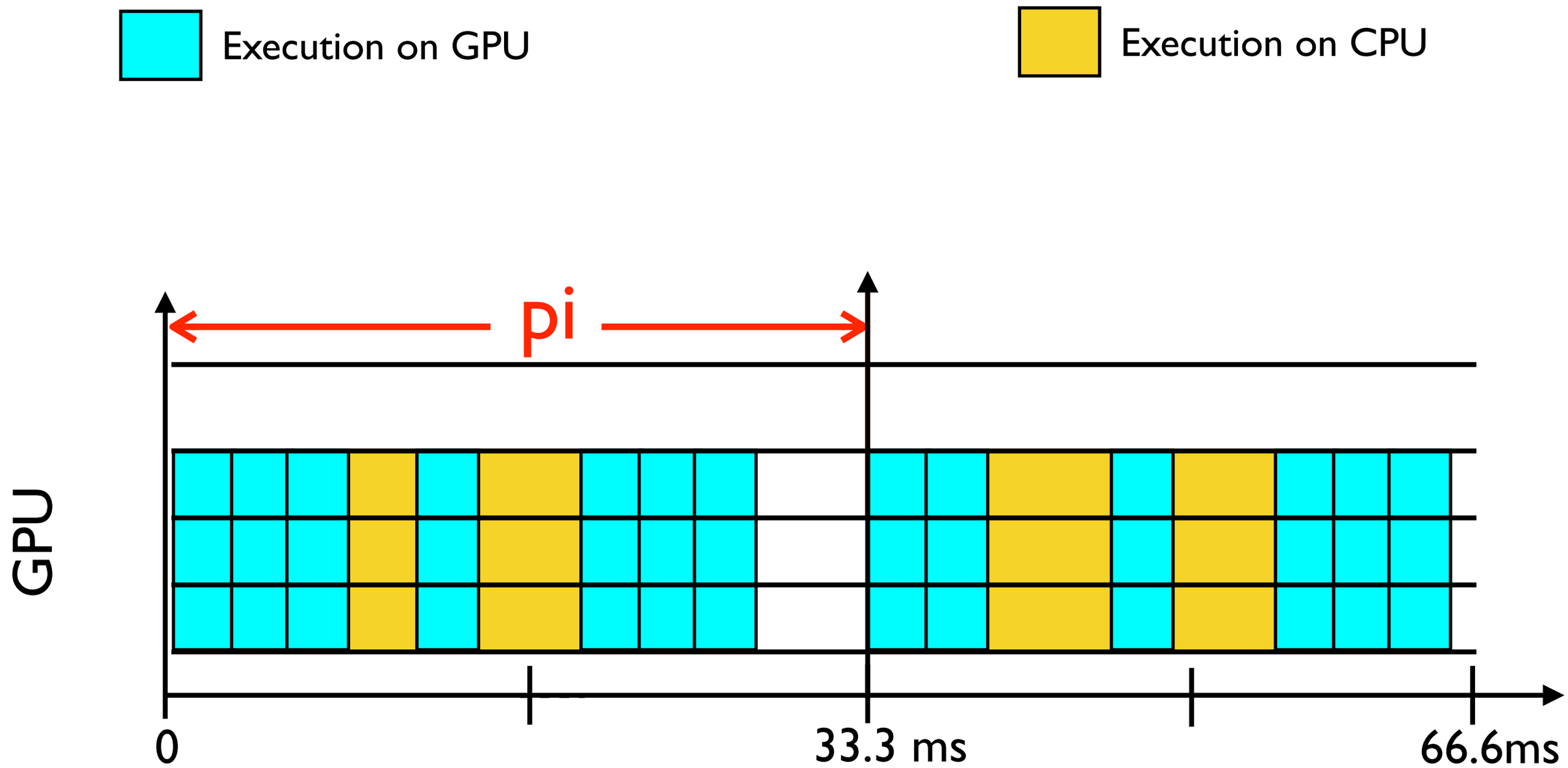

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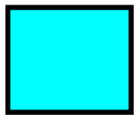
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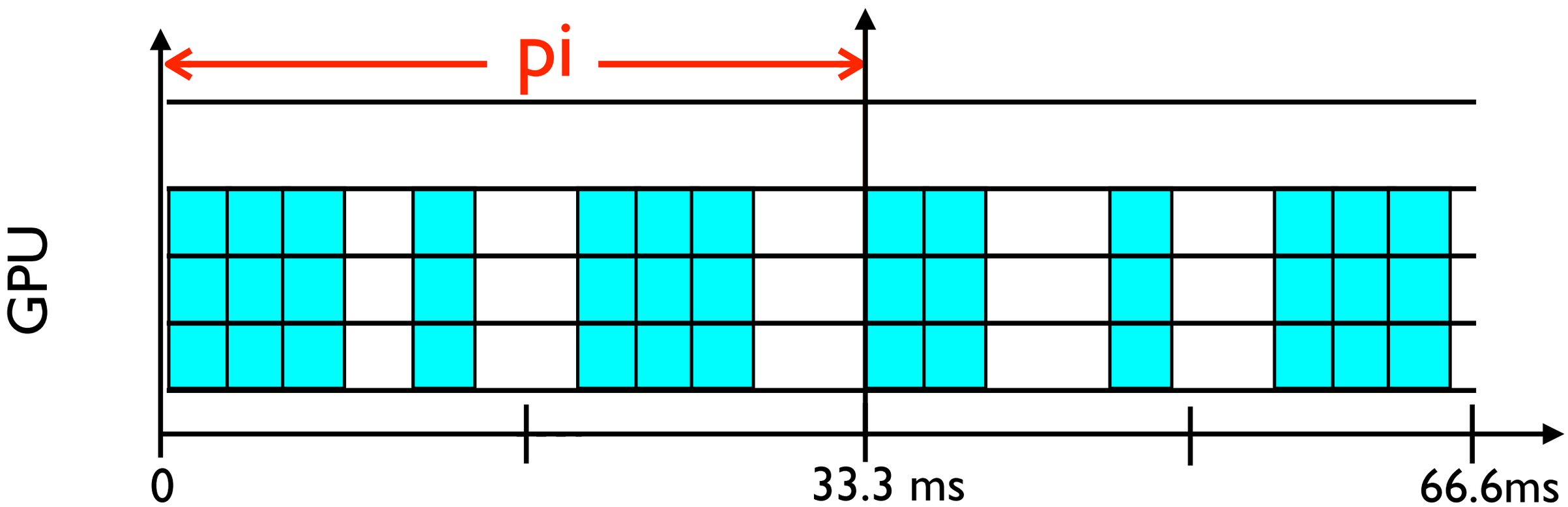

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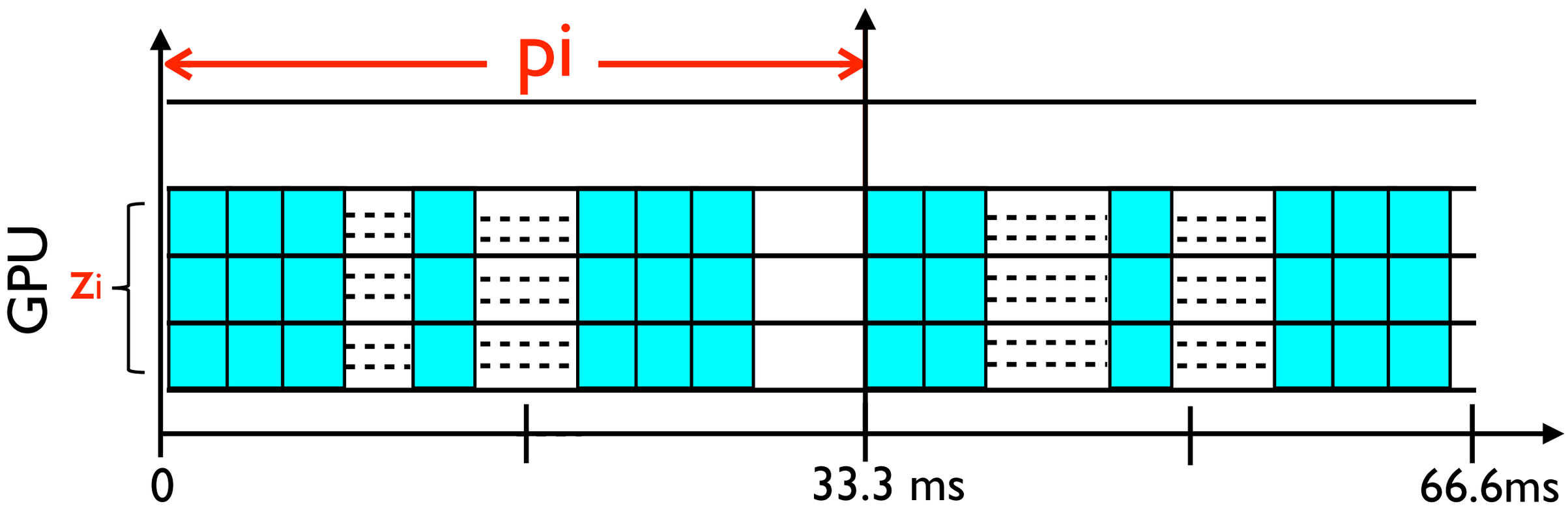

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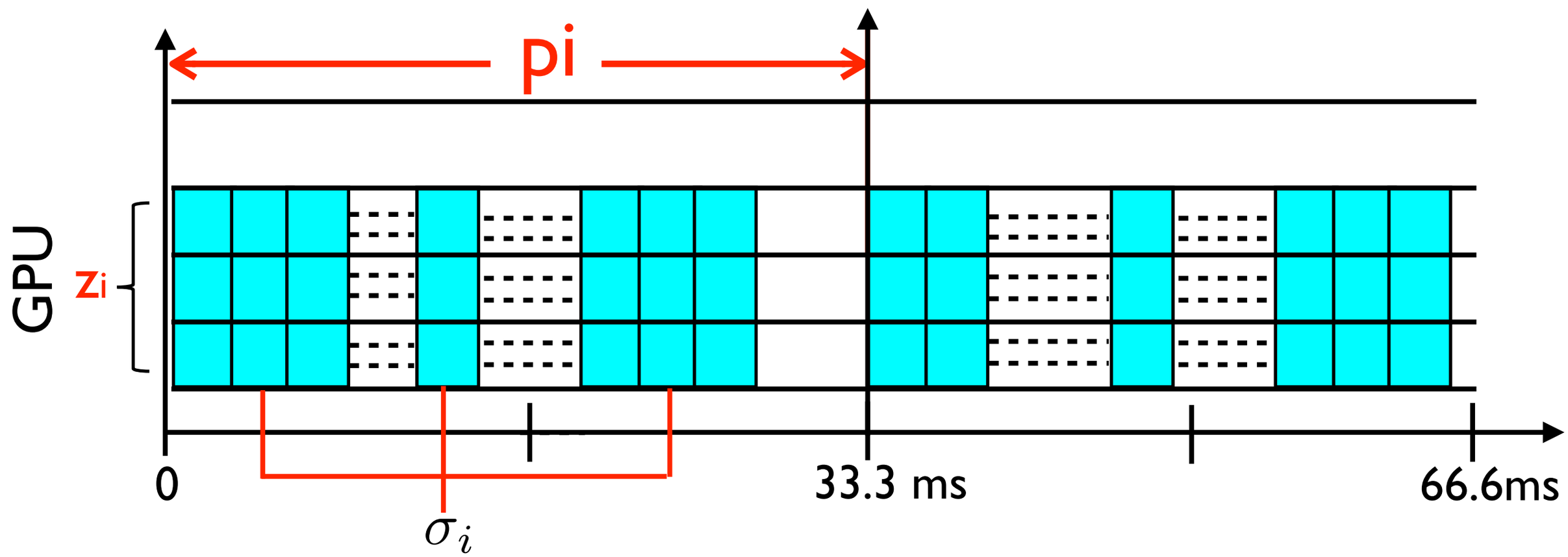

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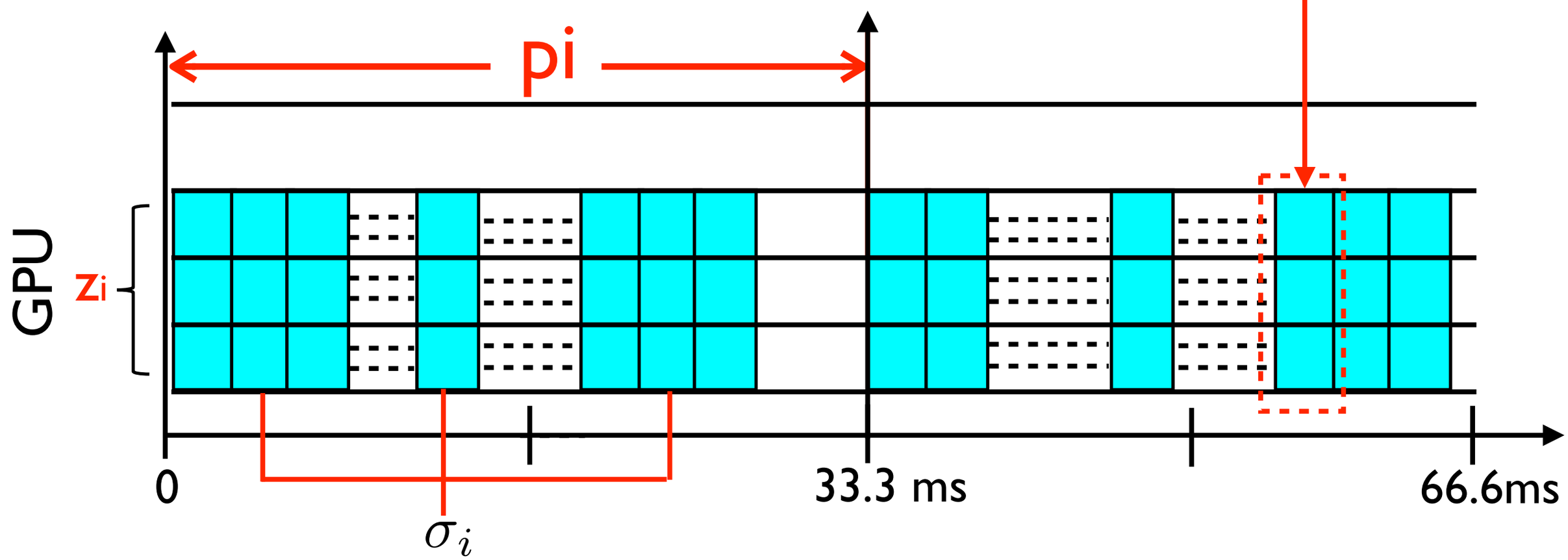

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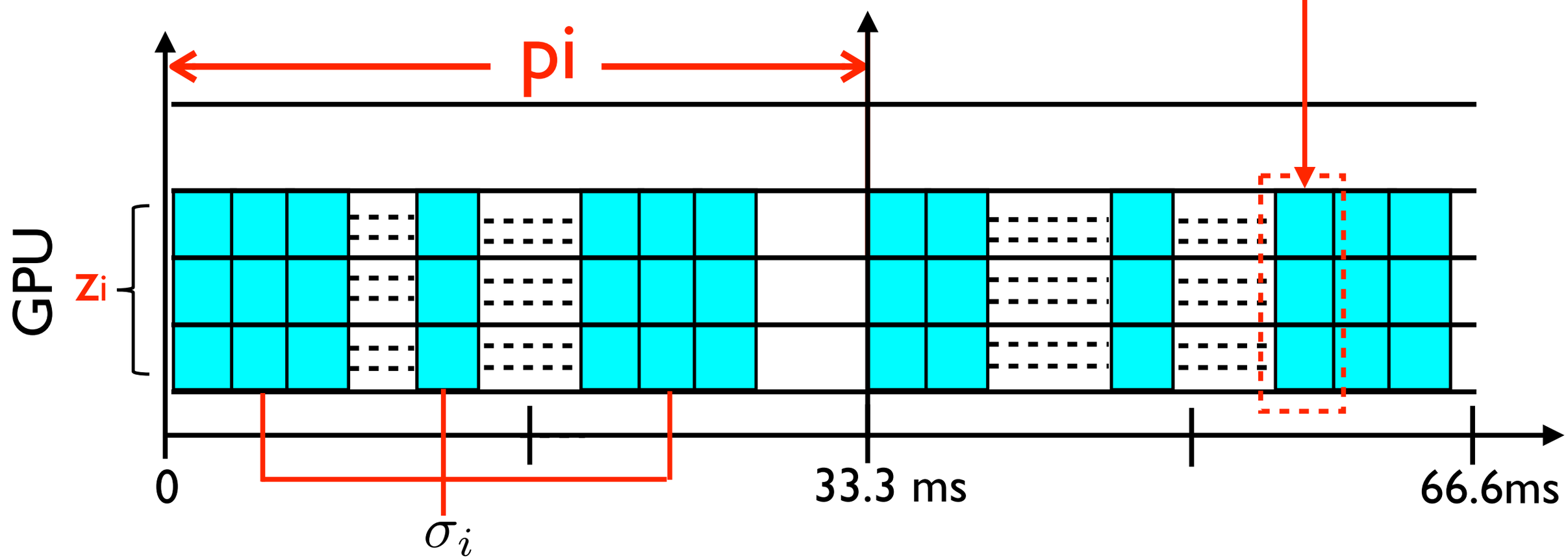
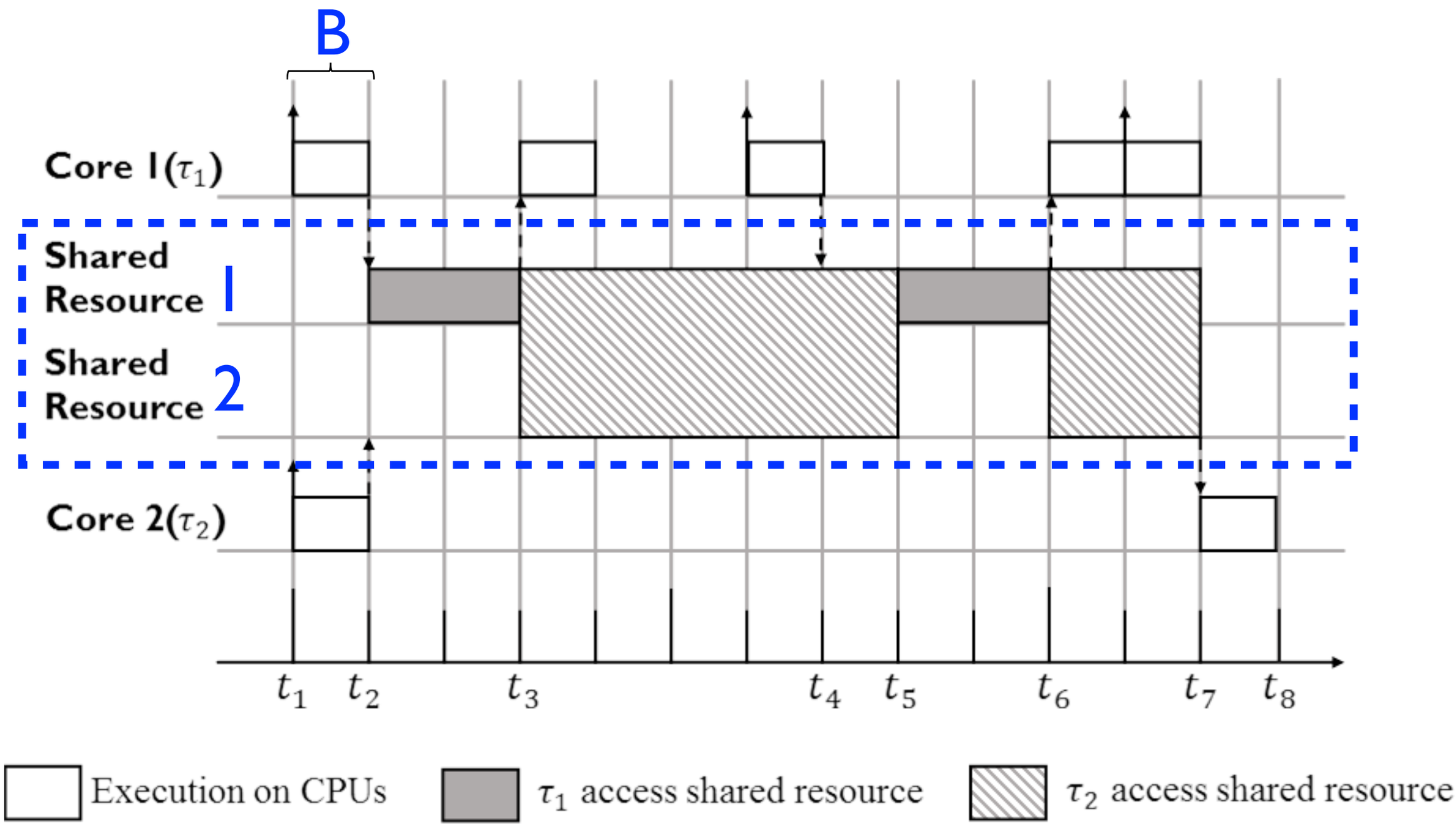


Rate-monotonic (RM) Scheduling on the shared resource.

Step 1: Developing schedulability tests for Limited preemptive suspending task scheduling



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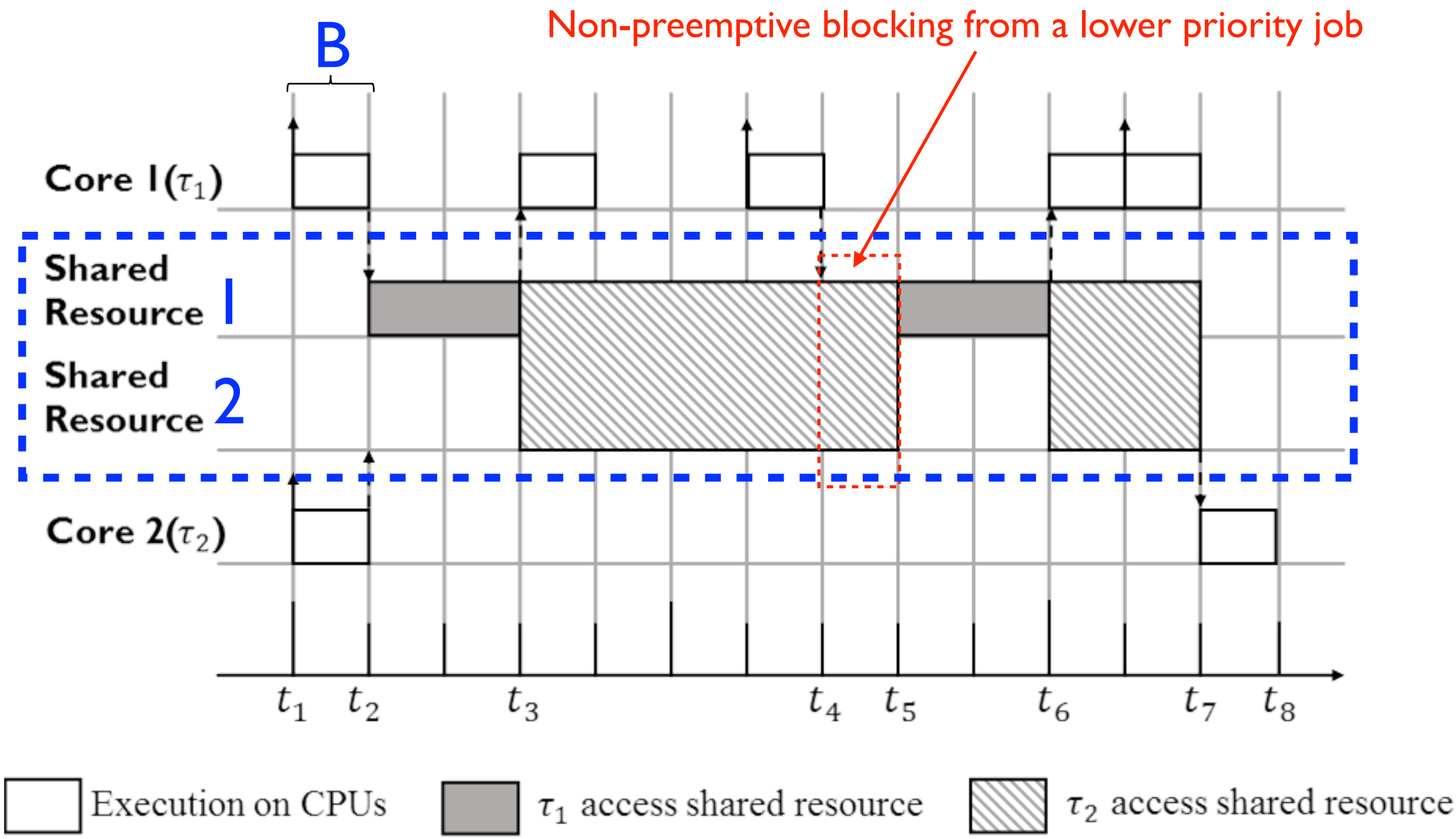

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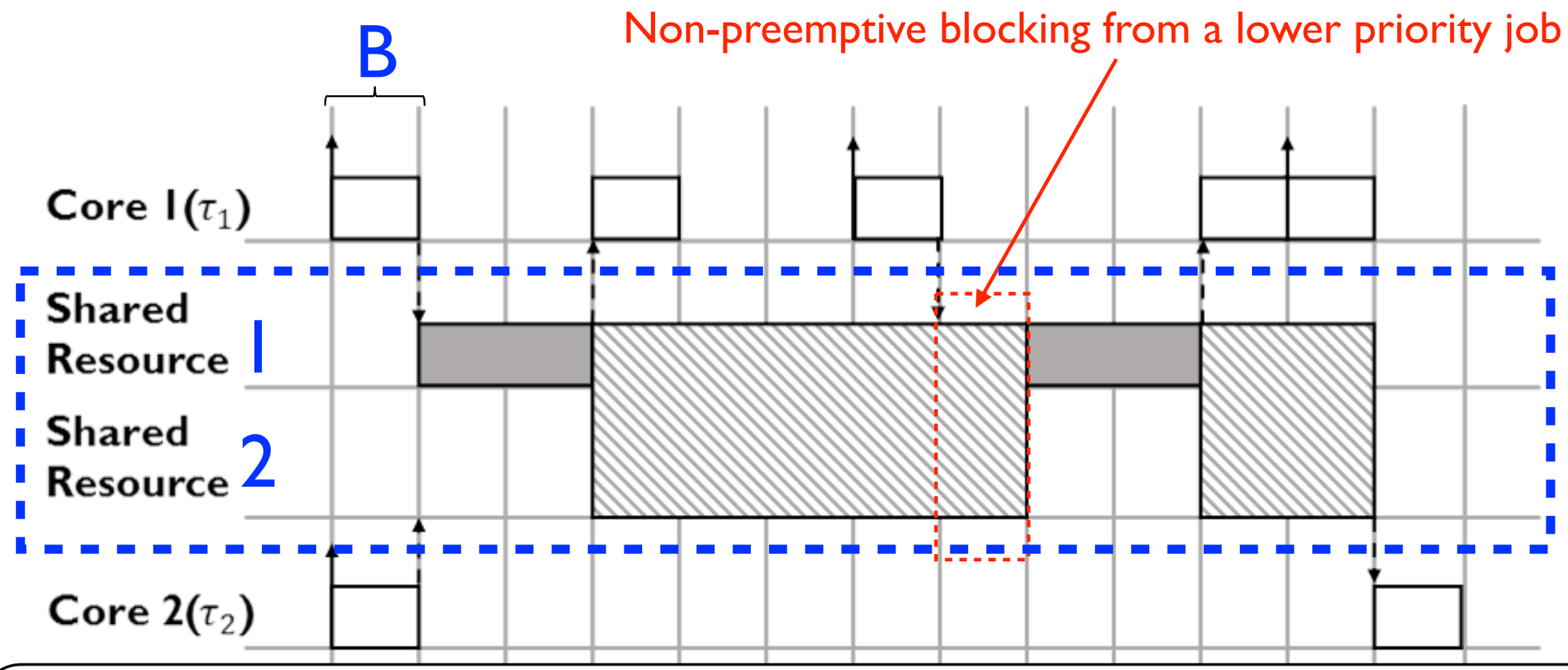

Observation

The maximum blocking time experienced by any job released by τ_i due to non-preemptive blocking on shared resource is at most σ_i times B.

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$$\exists t \mid 0 < t \leq p_k, \quad e_k + s_k + \sum_{\tau_i \in hp(k)} \left(\left\lceil \frac{t}{p_i} \right\rceil + 1 \right) s_i \leq t$$

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- Time-demand Analysis(TDA)
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Constant-time Tests (CT)

Lemma 5 (Suspension and Blocking as Carry-In).

$$\left(\frac{s_k + e_k + \sigma_k \times B}{p_k} + 2 \right) \prod_{\tau_i \in hp(k)} \left(\frac{s_i}{p_i} + 1 \right) \leq 3 \tag{5a}$$

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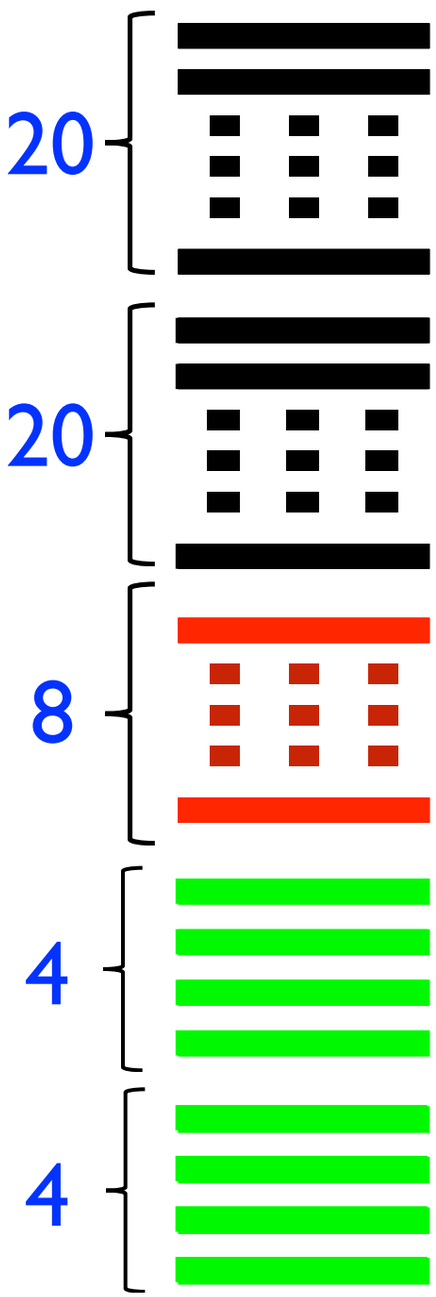
Step 2： Partitioning tasks onto a multi-unit shared resource



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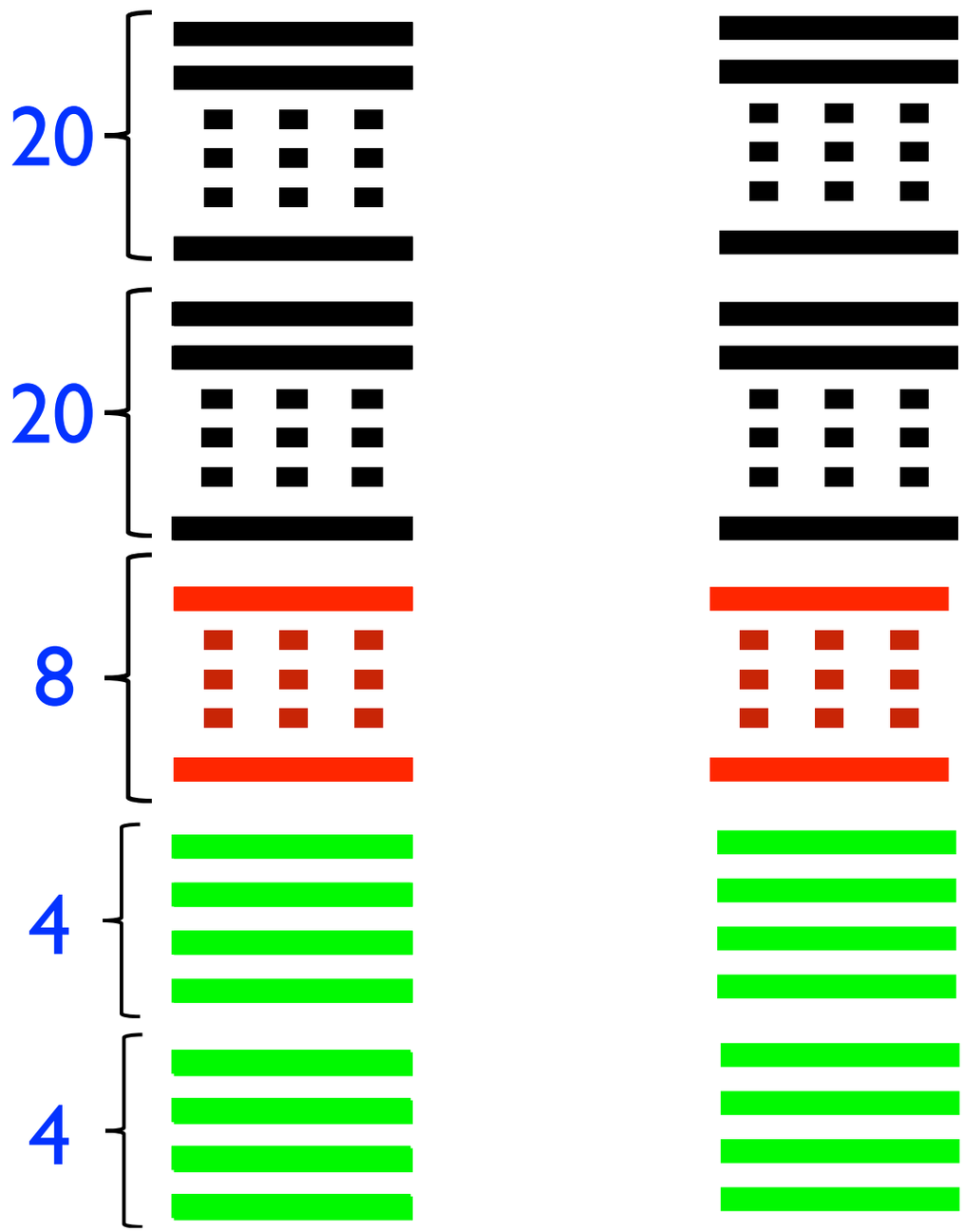

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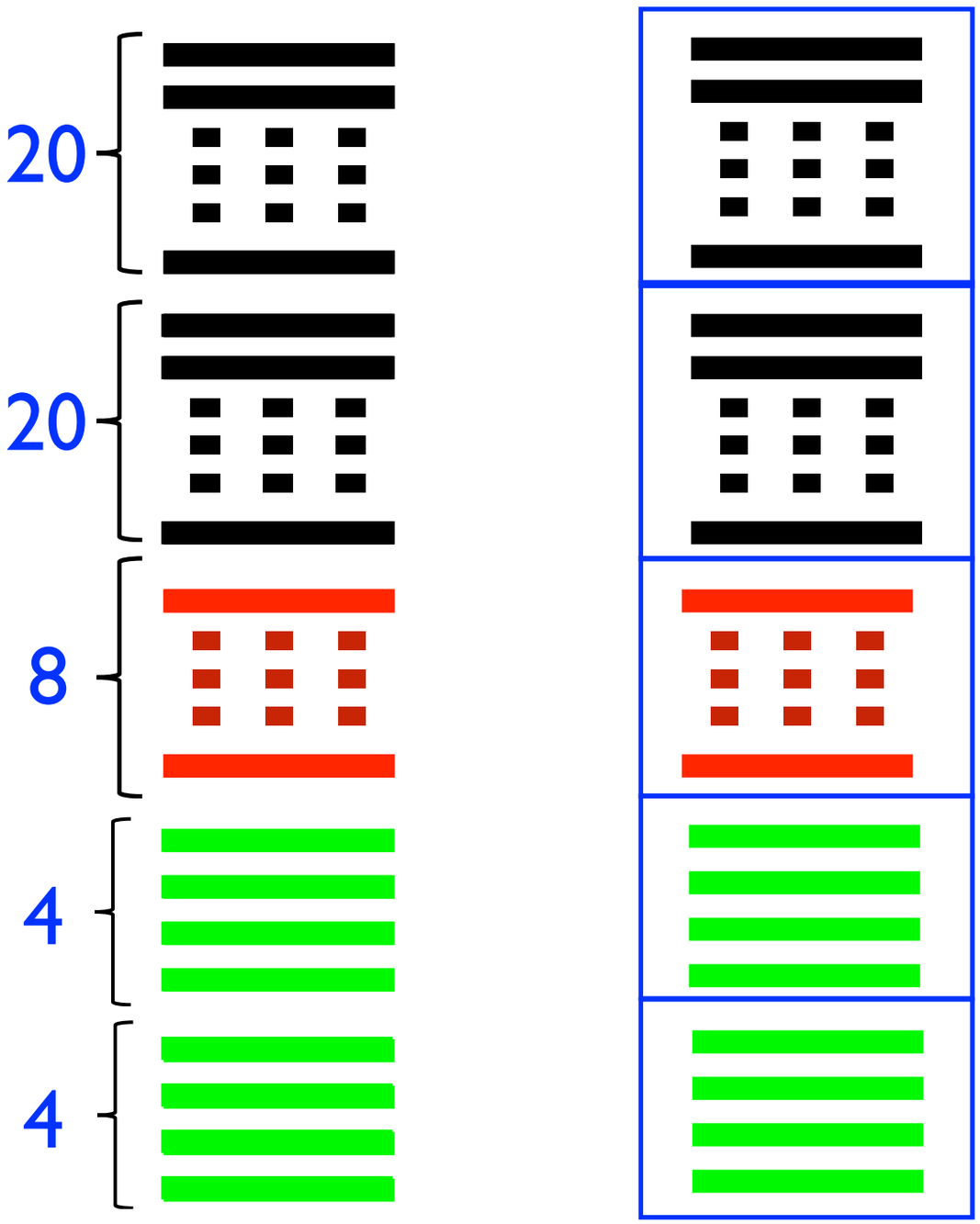

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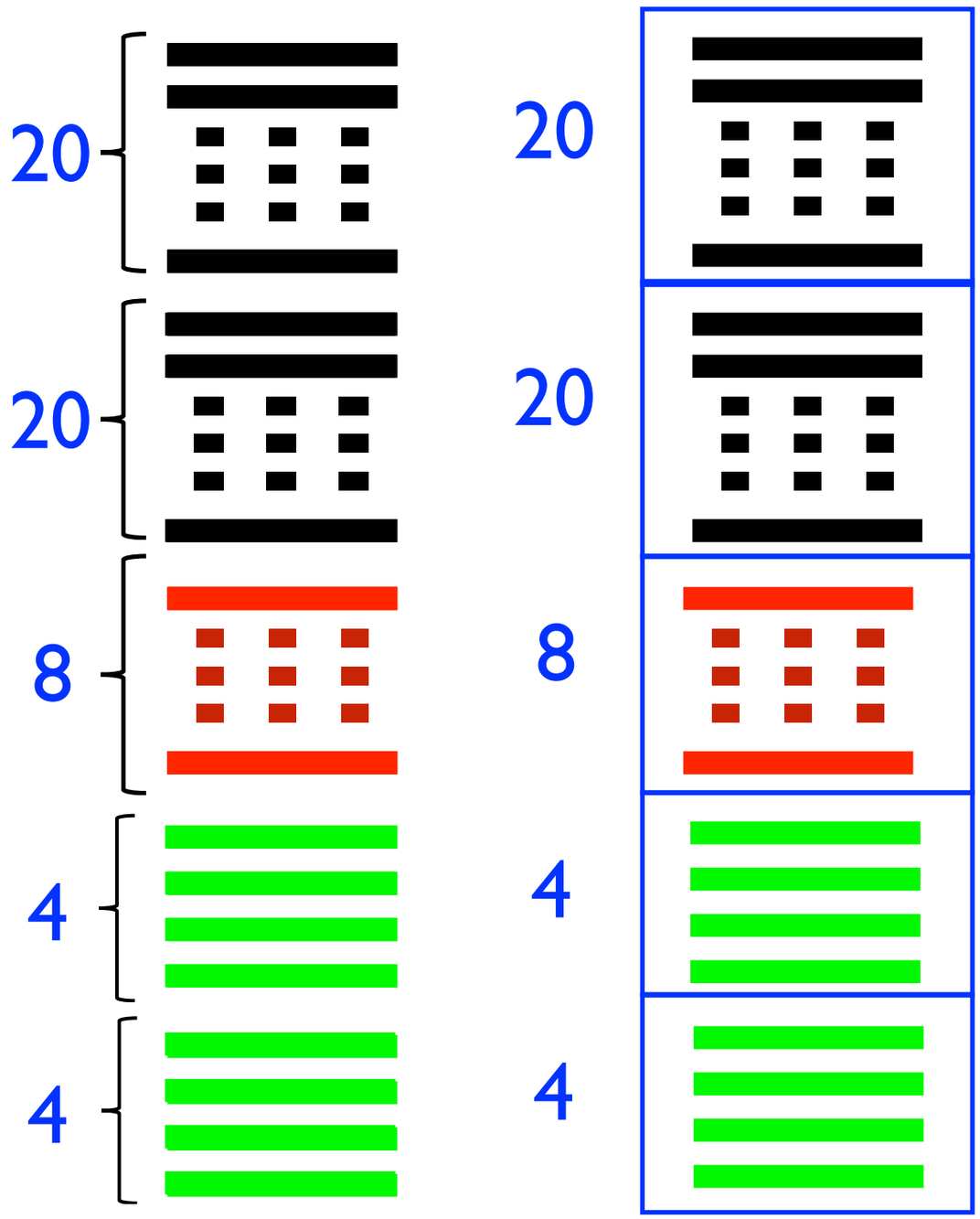

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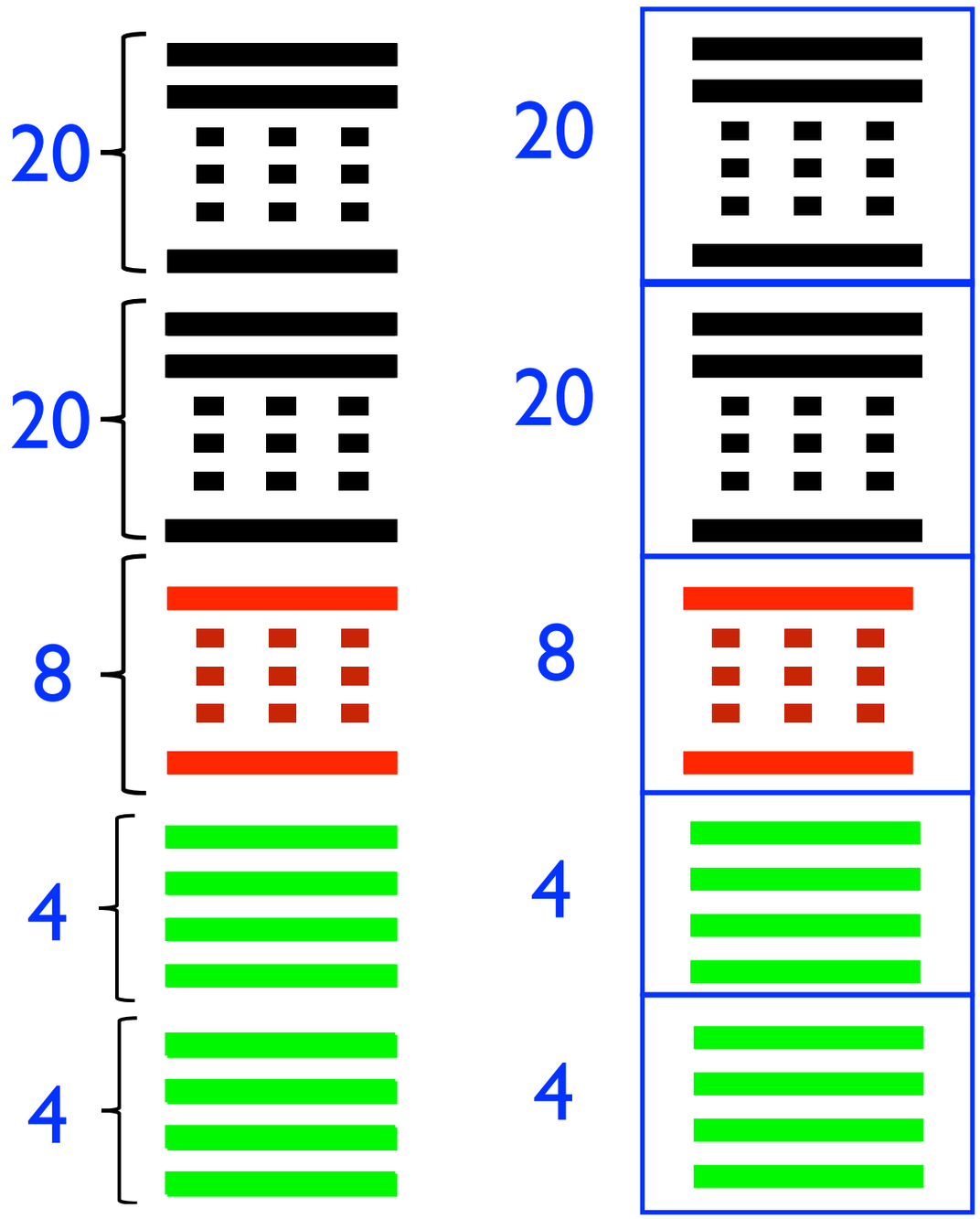

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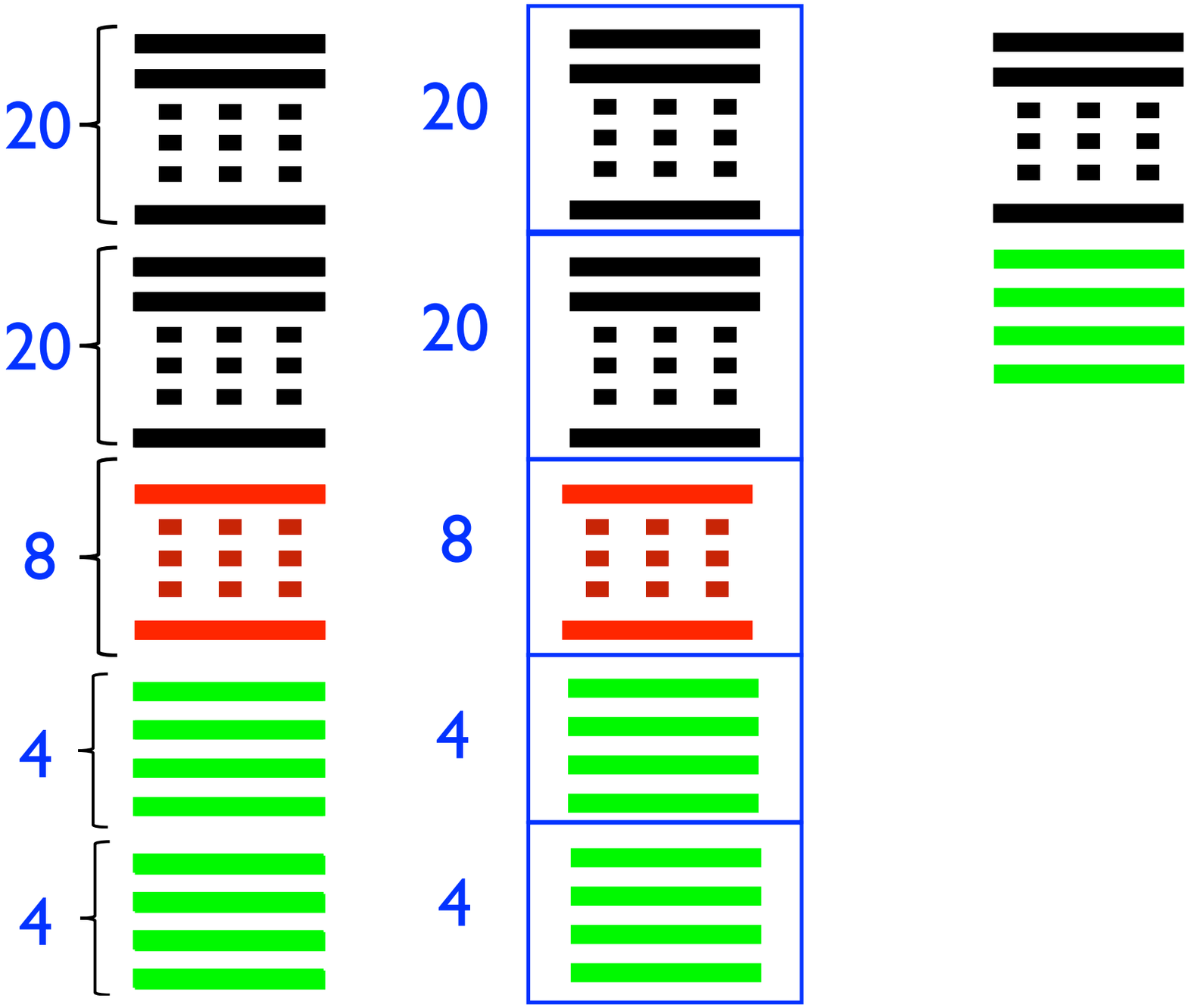

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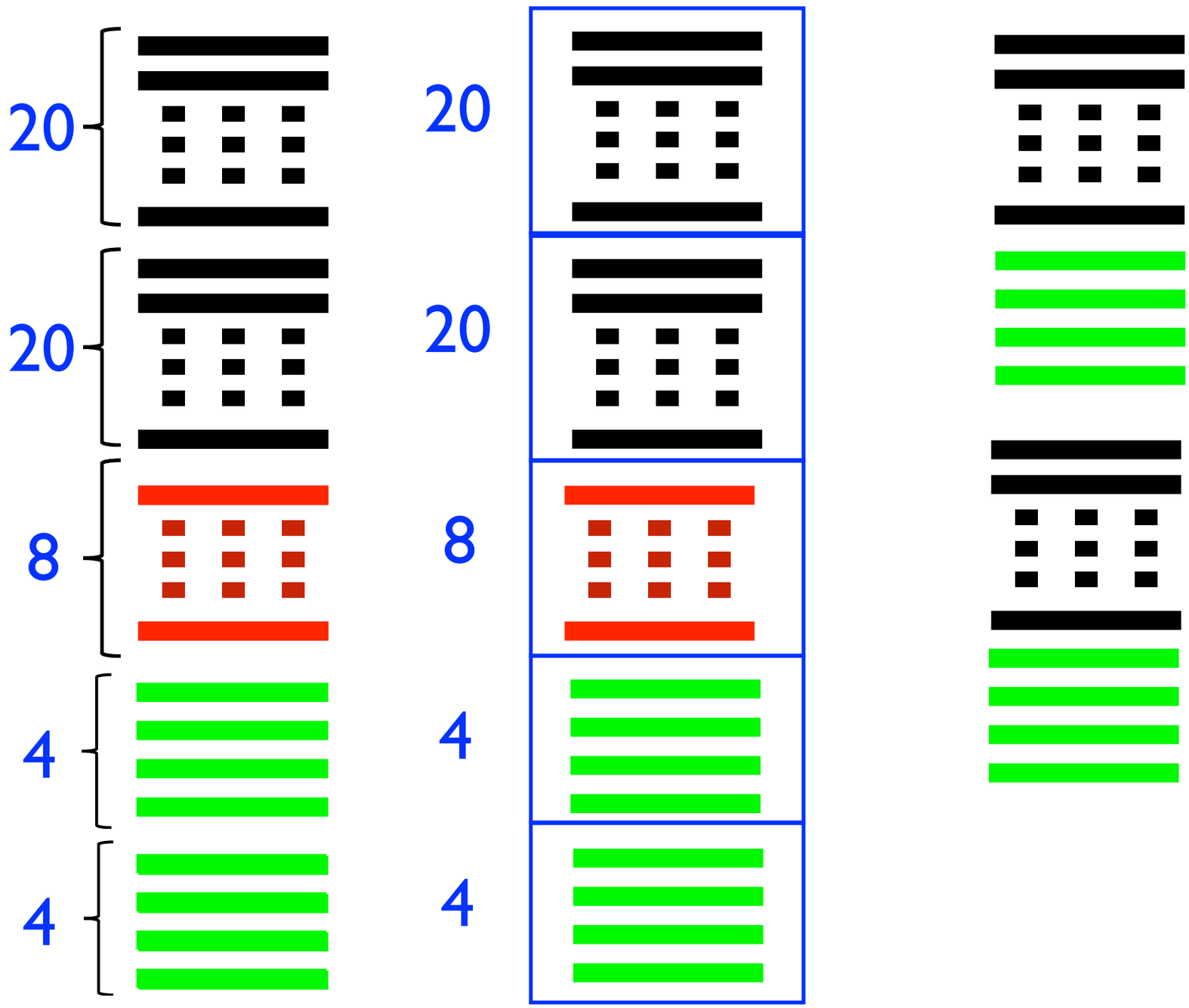

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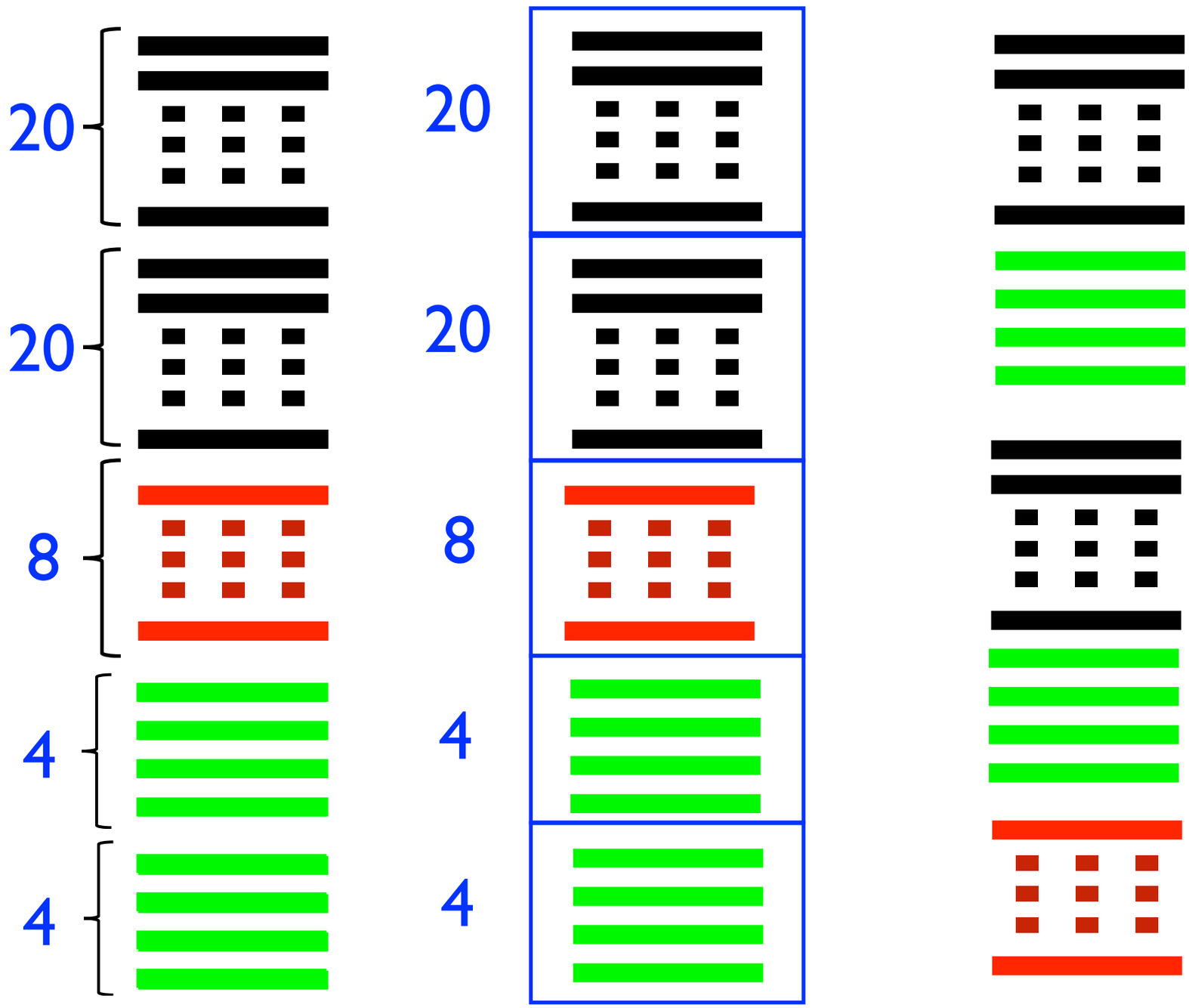

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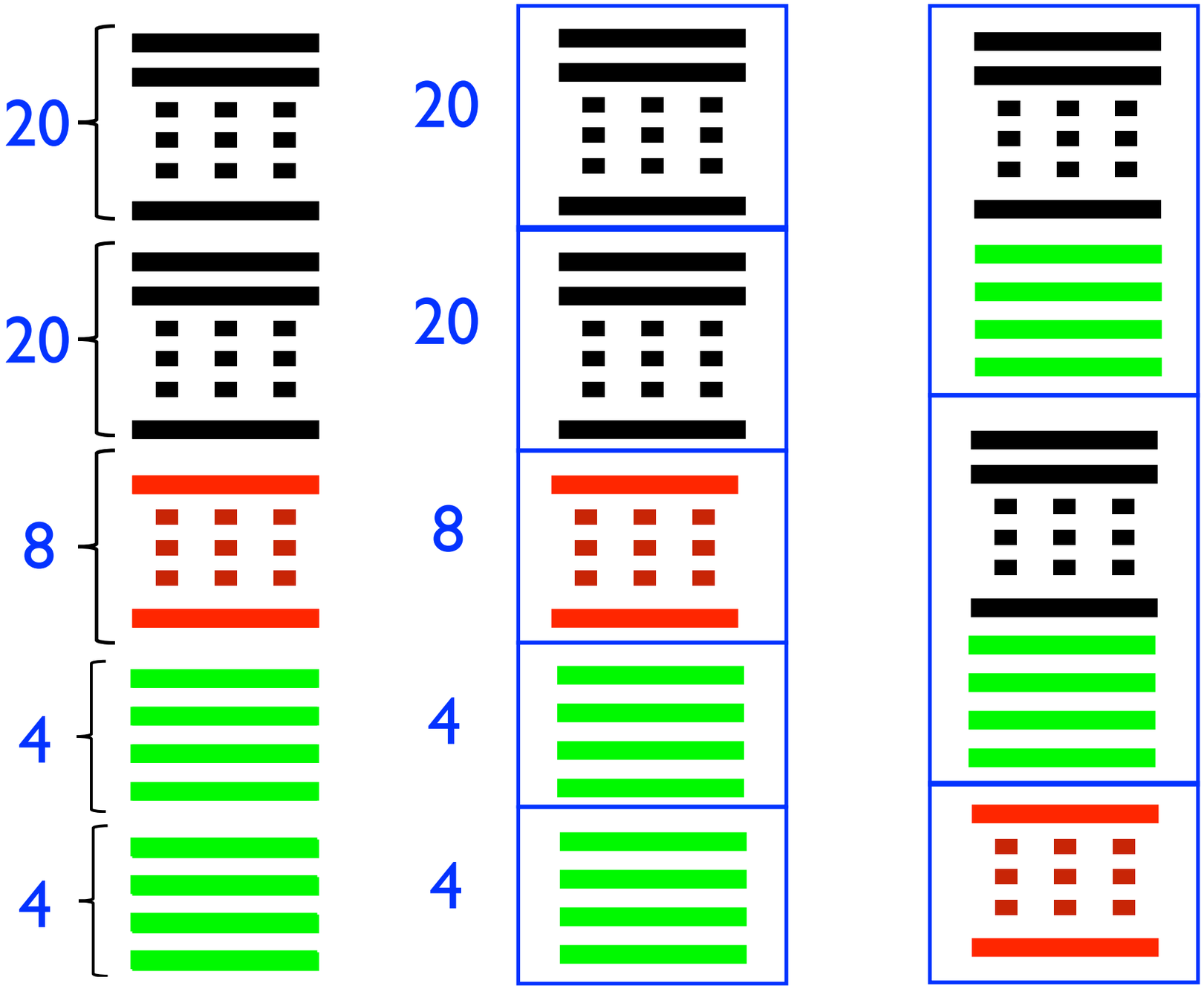

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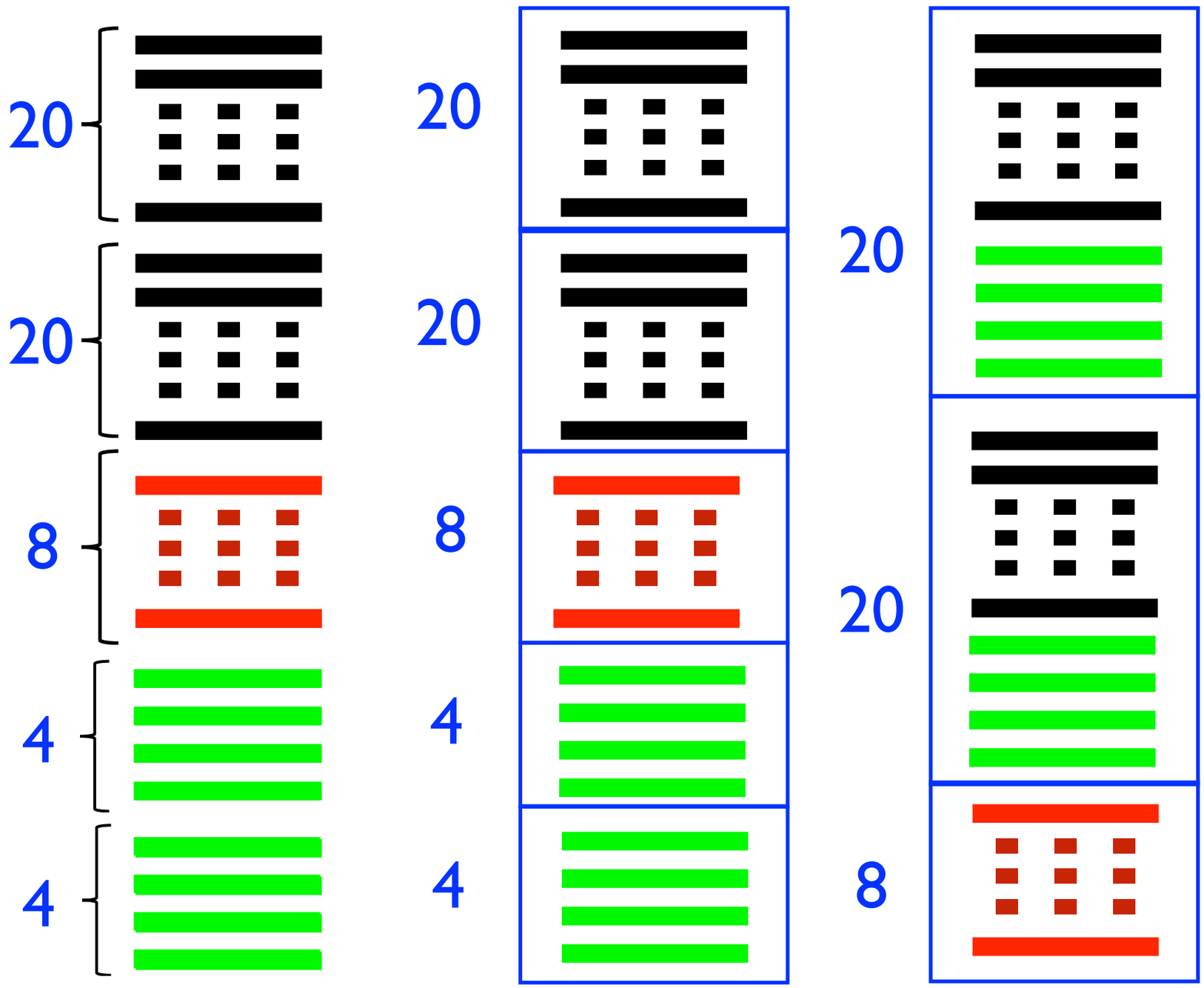

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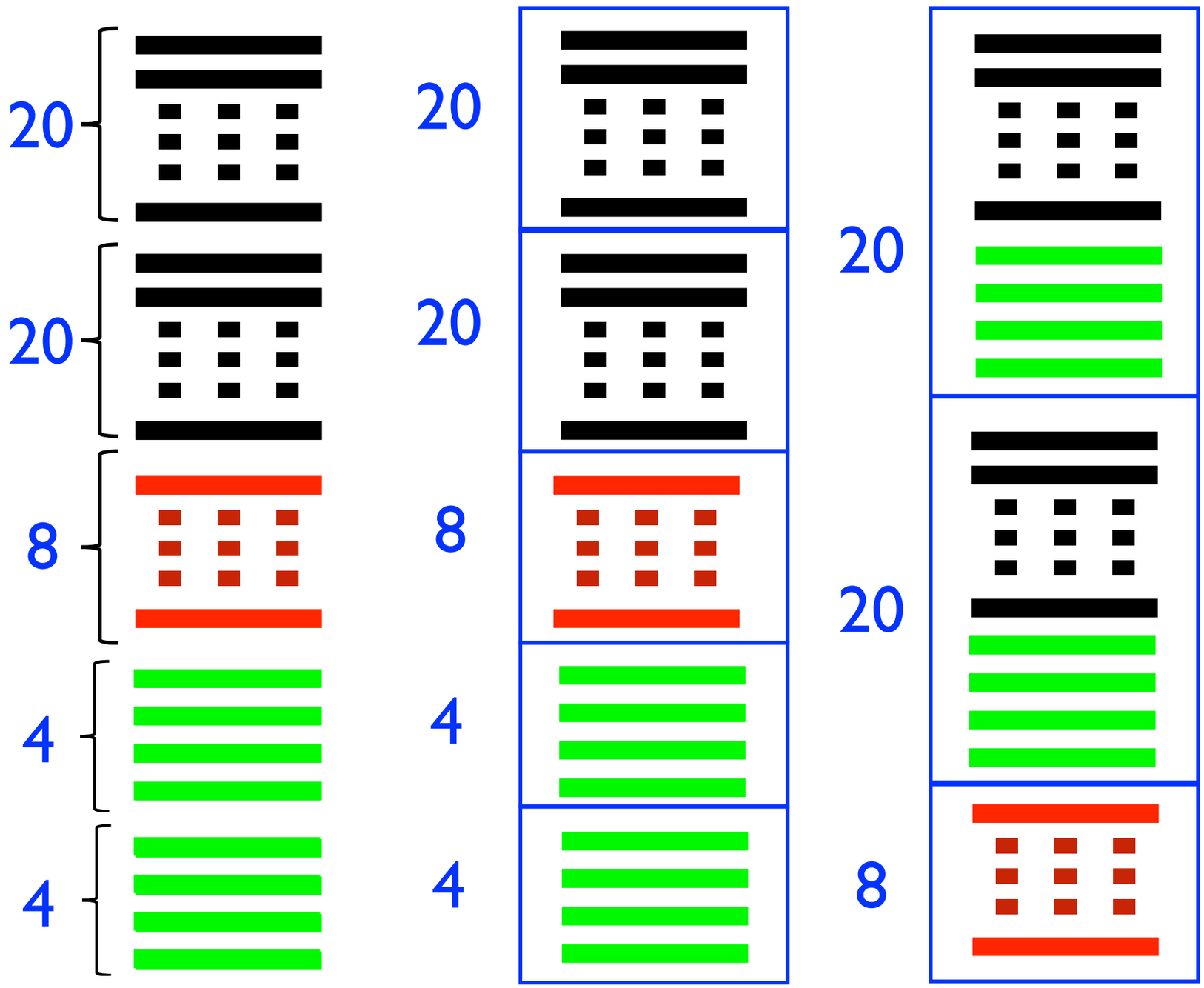

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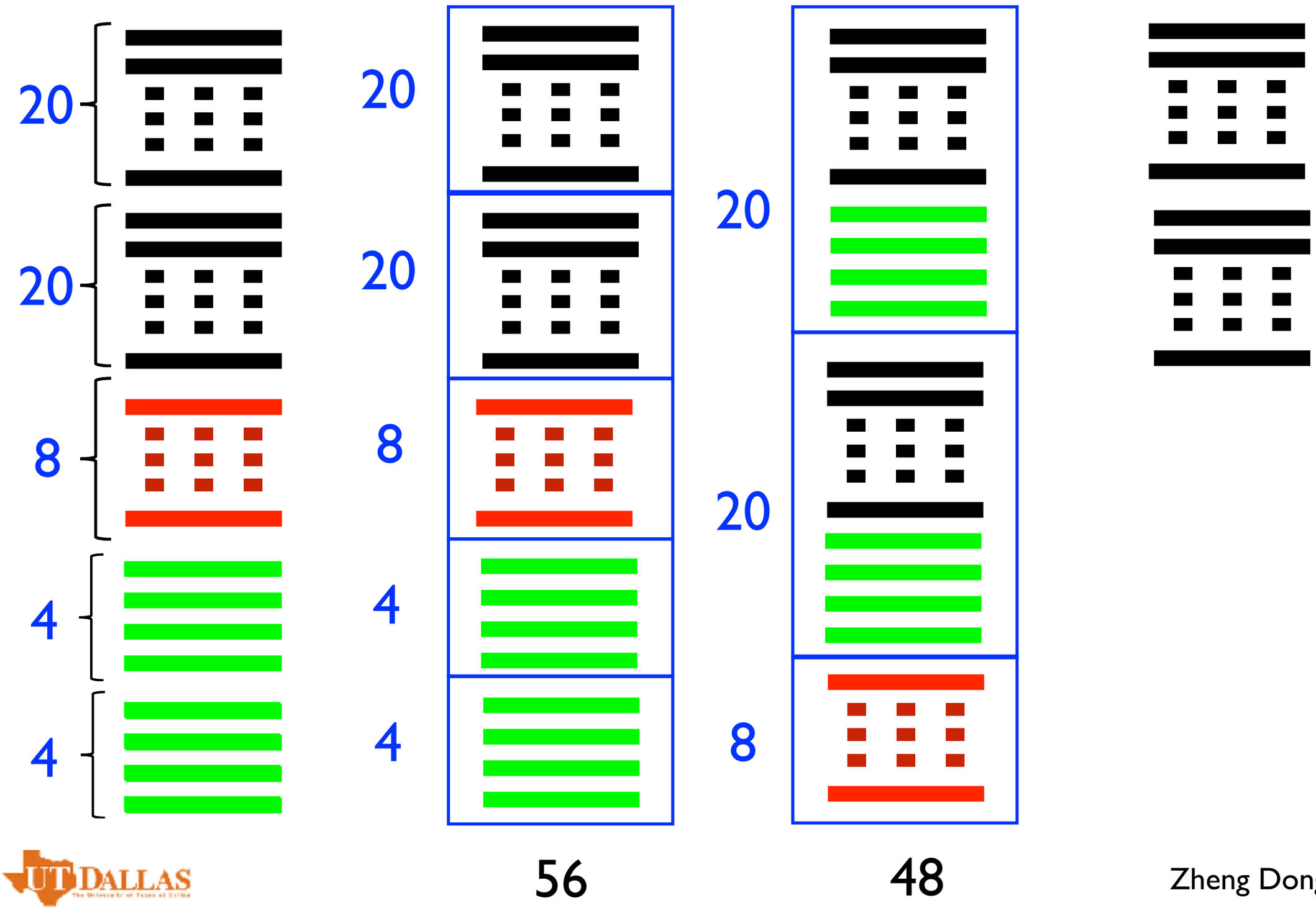
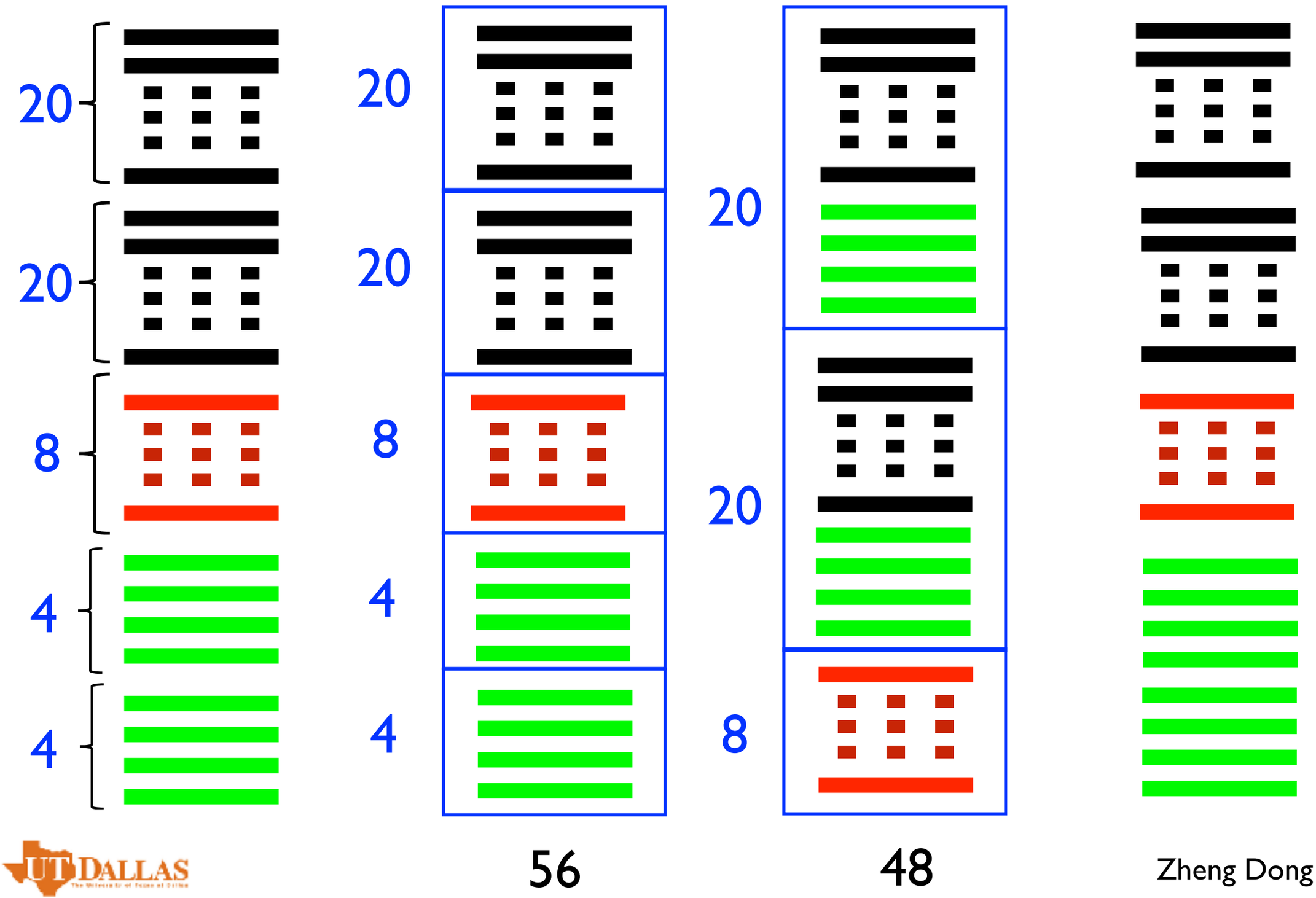


Step 2: Partitioning tasks onto a multi-unit shared resource



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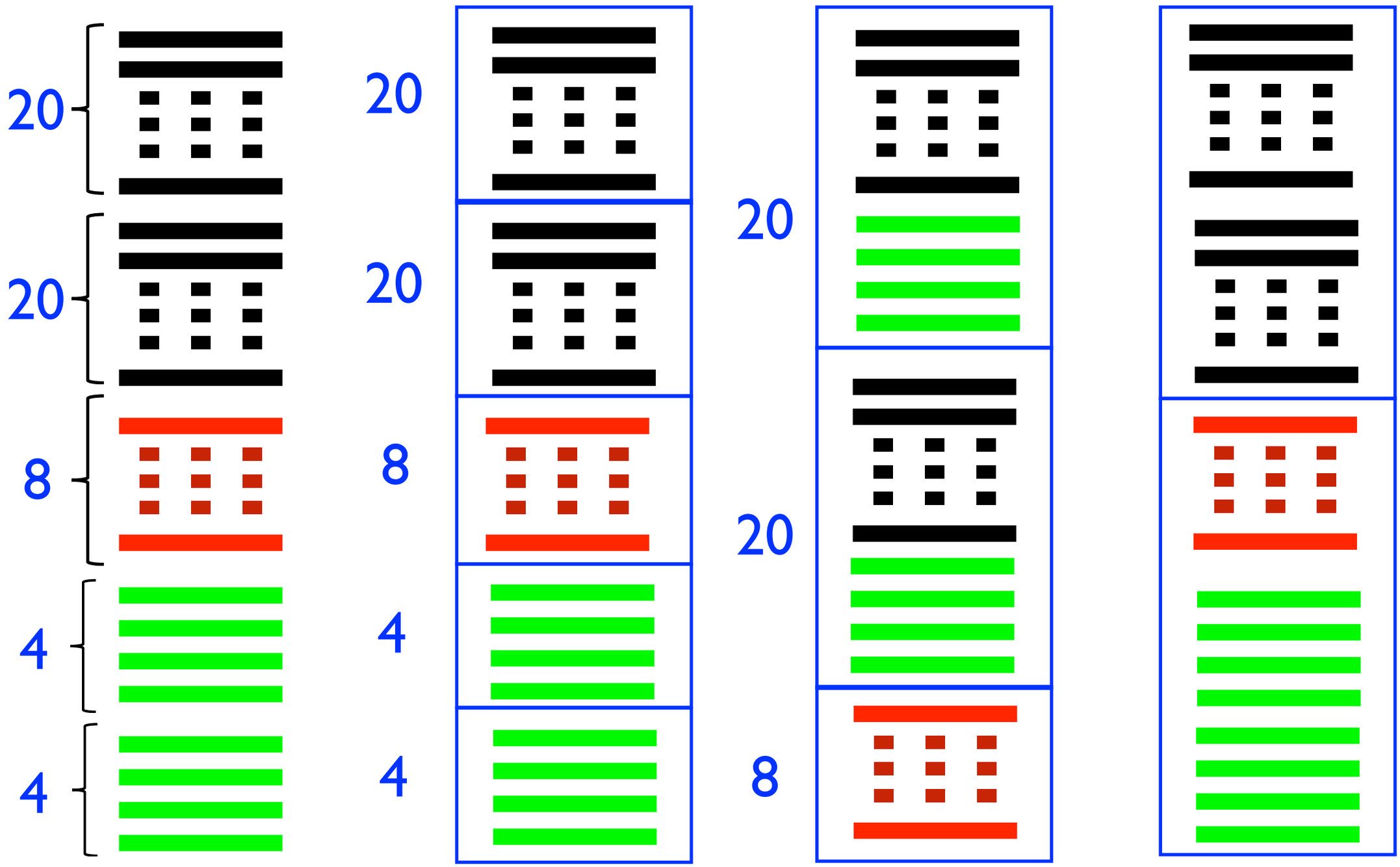

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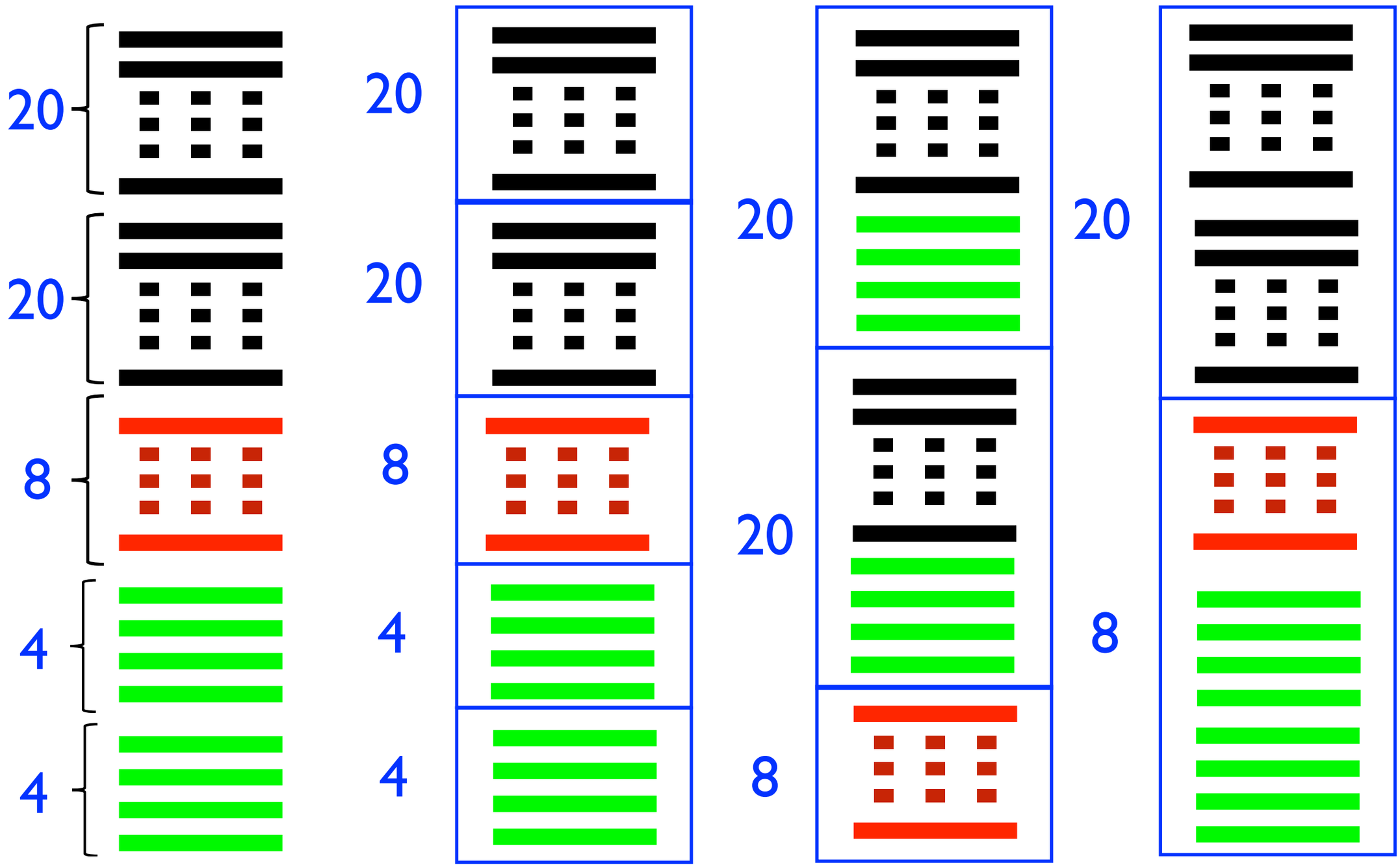

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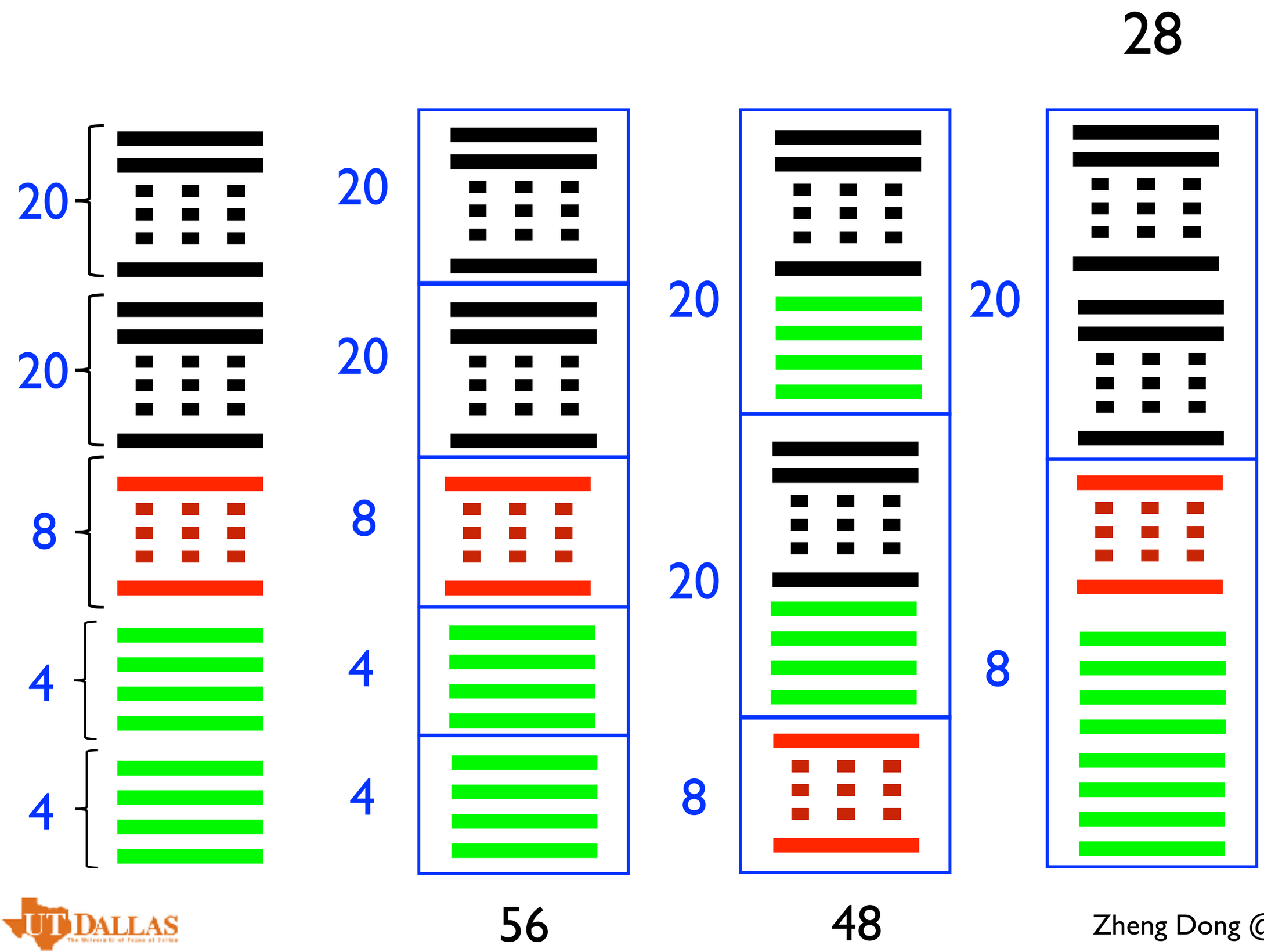

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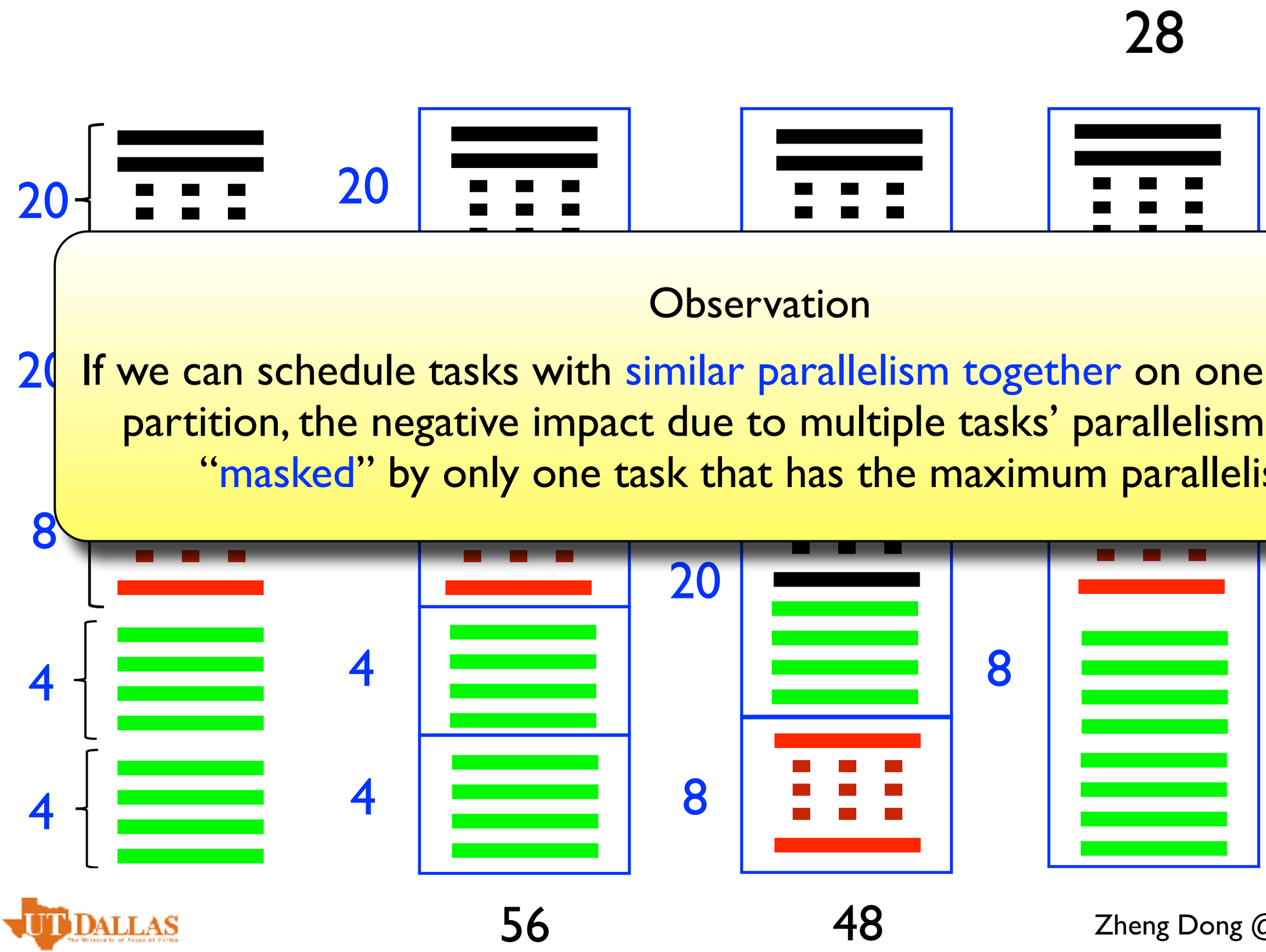


Observation

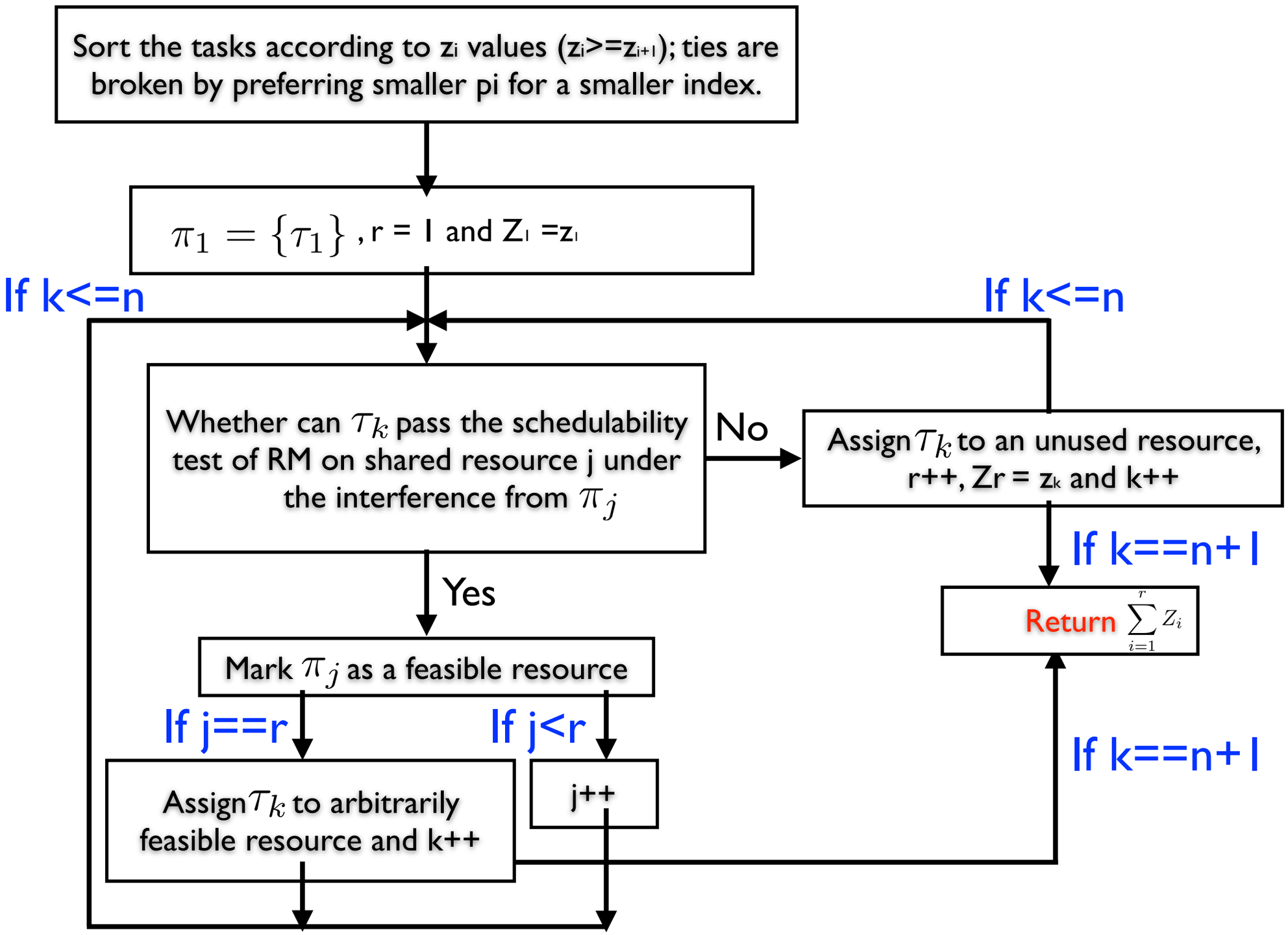
If we can schedule tasks with similar parallelism together on one resource partition, the negative impact due to multiple tasks' parallelism can be "masked" by only one task that has the maximum parallelism.

Step 2： Partitioning tasks onto a multi-unit shared resource

Step 2： Partitioning tasks onto a multi-unit shared resource

- Minimize the amount of required shared resource.
  - Developing a comprehensive set of partitioning algorithms combined with various schedulability tests.
  - Theoretically evaluate the effectivity of the proposed partitioning algorithms through upper bounding the system utilization loss.
  - Further improve the partitioning methods using an ILP-based approach.

Step 2： Partitioning tasks onto a multi-unit shared resource



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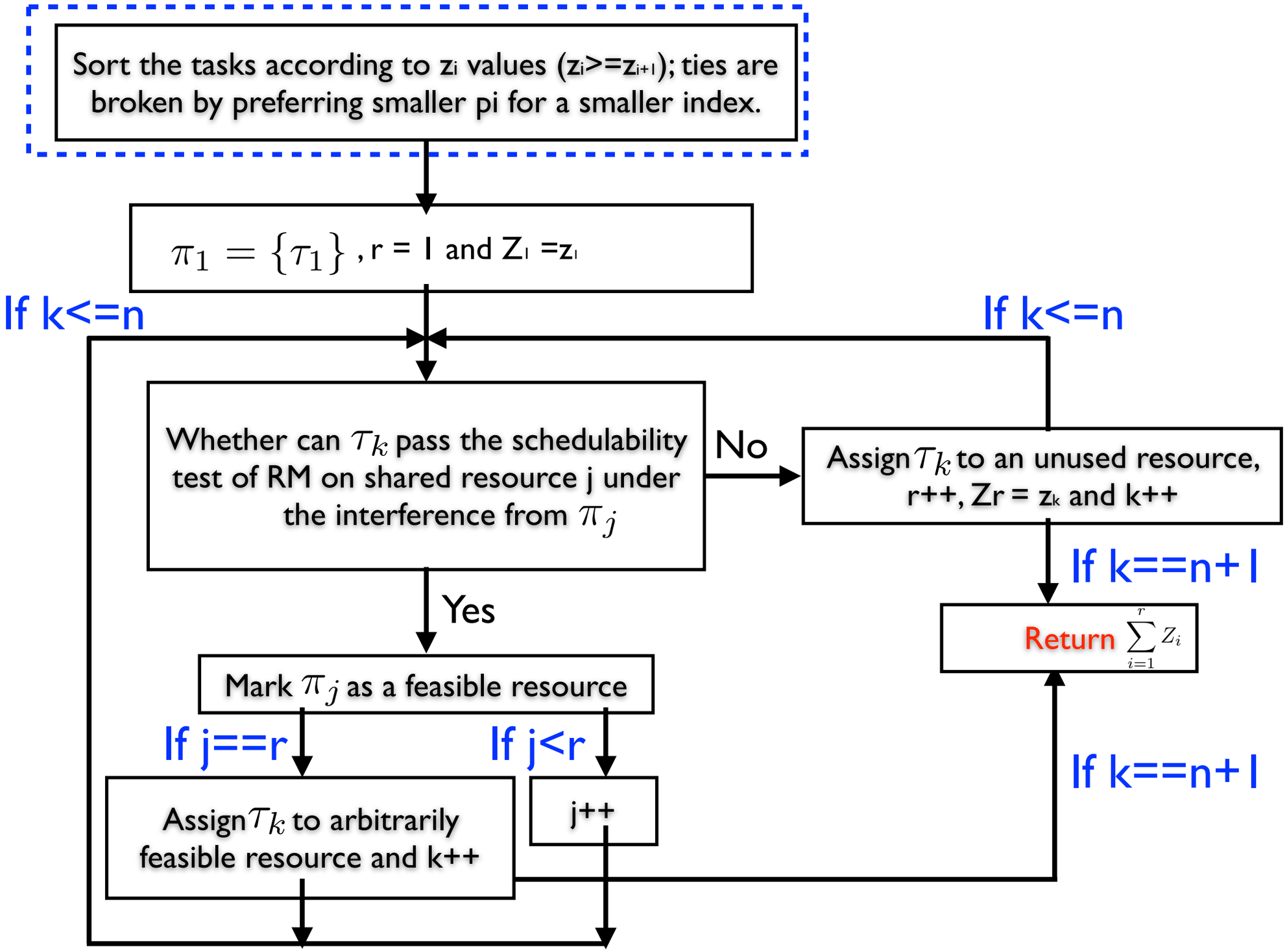

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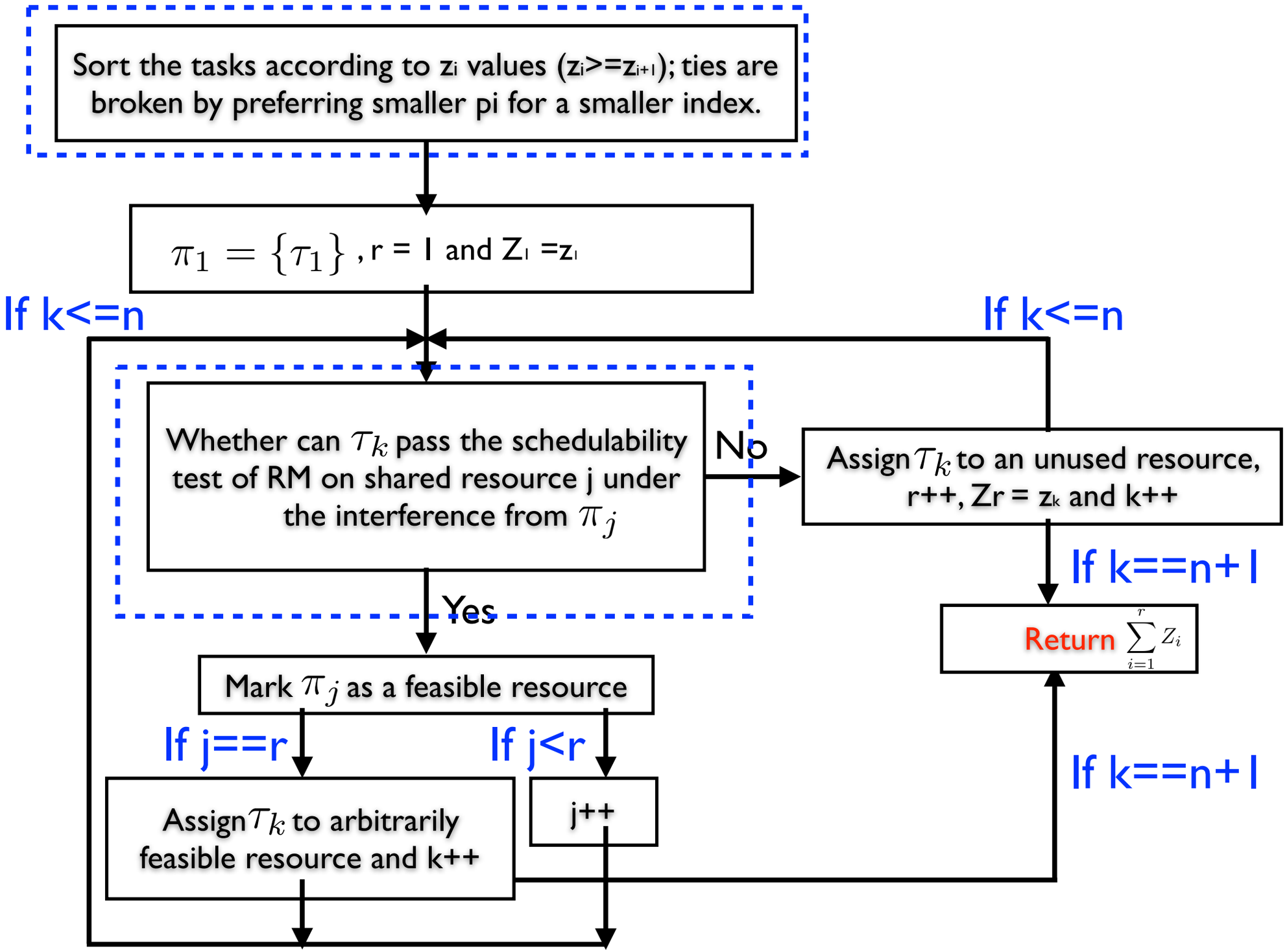

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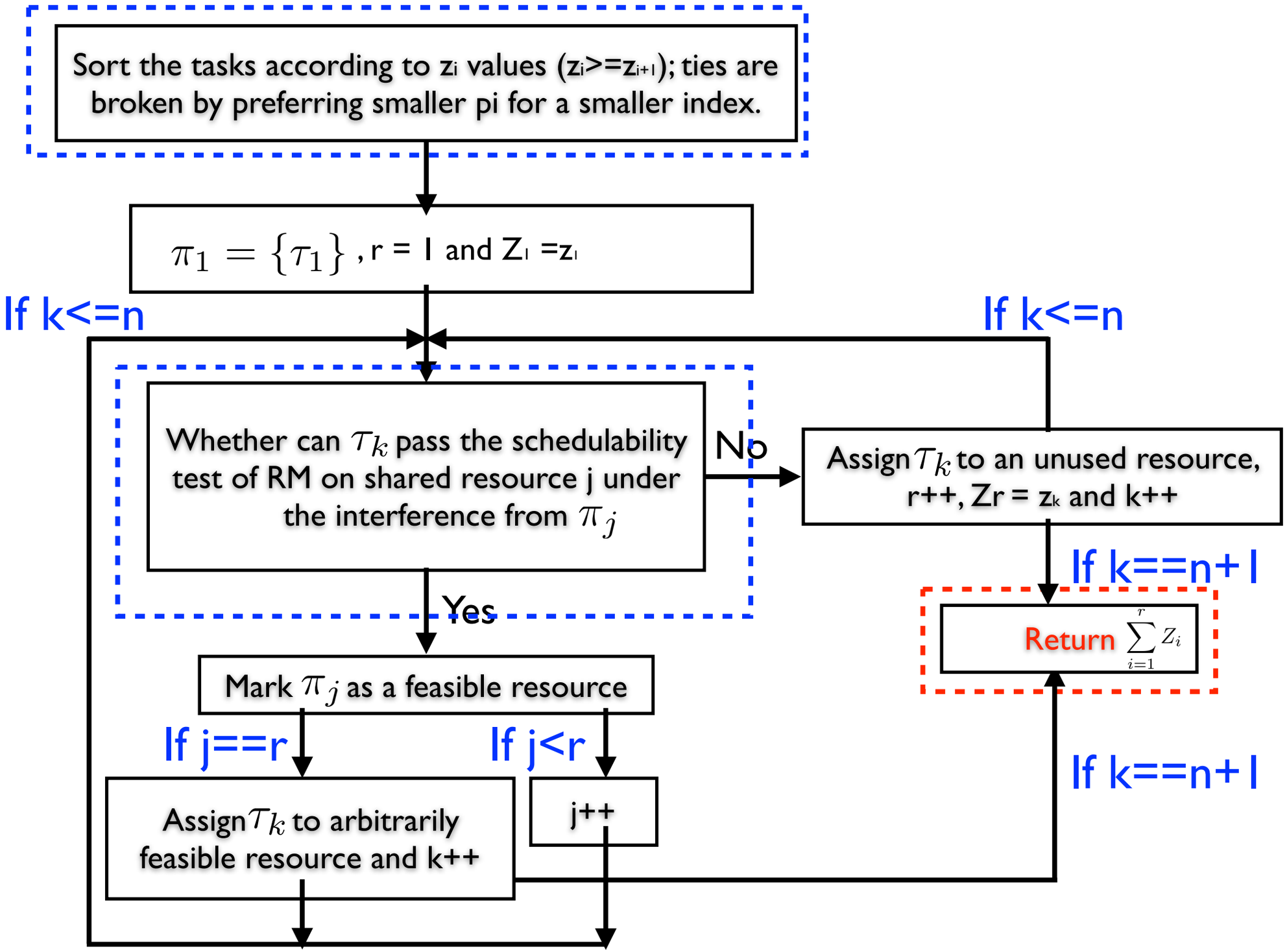

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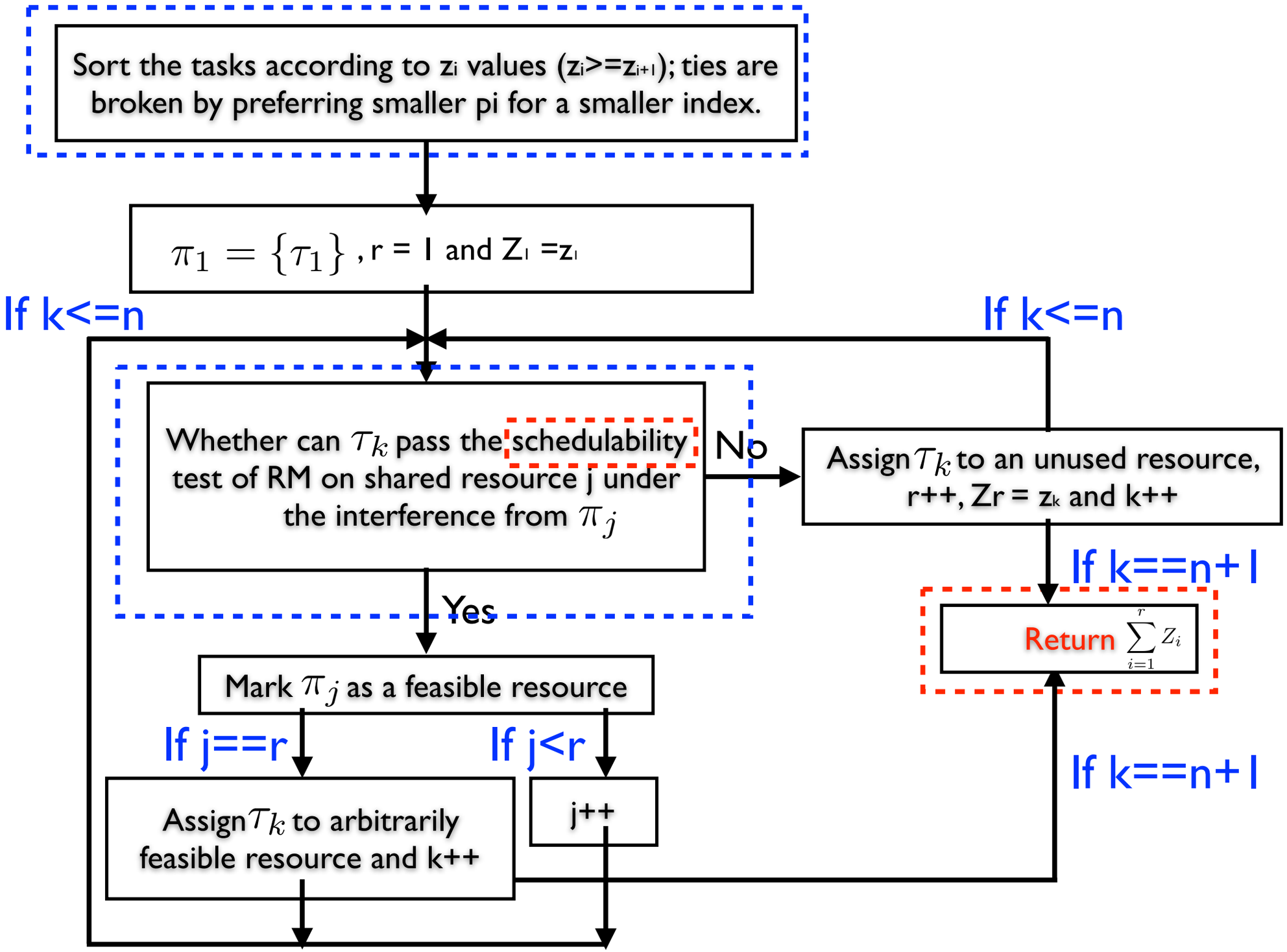

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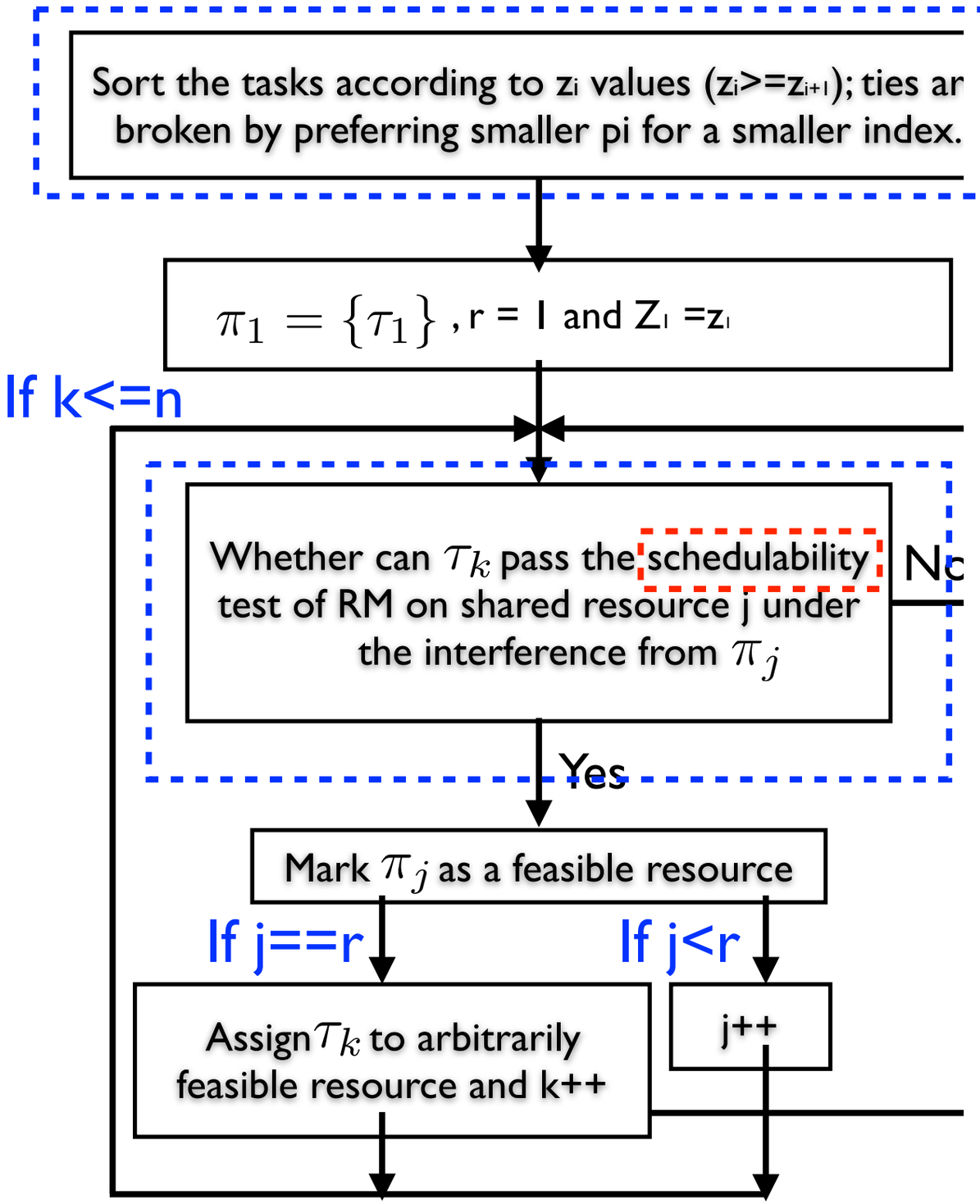

Time-demand Analysis(TDA)

Lemma 3 (Suspension and Blocking as Carry-In).

$$\exists t \mid 0 < t \le p_k, \quad s_k + e_k + \sigma_k \times B + \sum_{\tau_i \in hp(k)} \left(\left\lceil \frac{t}{p_i} \right\rceil + 1 \right) s_i \le t \tag{3}$$

Lemma 4 (Suspension and Blocking as Jitter).

$$\exists t \mid 0 < t \le p_k, \quad s_k + e_k + \sigma_k \times B + \sum_{\tau_i \in hp(k)} \left\lceil \frac{t + p_i - s_i}{p_i} \right\rceil s_i \le t$$

Constant-time Tests (CT)

Lemma 5 (Suspension and Blocking as Carry-In).

$$\left(\frac{s_k + e_k + \sigma_k \times B}{p_k} + 2 \right) \prod_{\tau_i \in hp(k)} \left(\frac{s_i}{p_i} + 1 \right) \le 3 \tag{5a}$$

$$\sum_{\tau_i \in hp(k)} \frac{s_i}{p_i} \le \ln \left(\frac{3}{\frac{s_k + e_k + \sigma_k \times B}{p_k} + 2} \right) \tag{5b}$$

Lemma 6 (Suspension and Blocking as Jitter).

$$\frac{e_k + s_k + \sigma_k \times B}{p_k} + \frac{\sum_{\tau_i \in hp(k)} \left(2 s_i - \frac{s_i^2}{p_i} \right)}{p_k} + \sum_{\tau_i \in hp(k)} \frac{s_i}{p_i} \le 1 \tag{6}$$

Step 2: Partitioning tasks onto a multi-unit shared resource

Upper bounding the system utilization loss

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ST-FF-CT(Carry)

$$U_{sum} \leq m \times \ln\left(\frac{3}{2 + \overline{u_{max}}}\right)$$

$$where\ \overline{u_{max}} = \max_{i=m+1}^{n} \frac{e_i + s_i + \sigma_i \times B}{p_i}$$

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$$U_{sum} \leq \frac{m}{3}(1 - \overline{u_{max}})$$

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Step 2： Partitioning tasks onto a multi-unit shared resource

Upper bounding the system utilization loss

ST-FF-CT(Carry)

$$U_{sum} \leq m \times \ln\left(\frac{3}{\ldots}\right)$$

Intuitively, task partitioning methods always suffer from both scheduling algorithms and bin-packing induced utilization loss.

$$\frac{e_k + s_k + \sigma_k \times B}{p_k} + \frac{\sum_{\tau_i \in hp(k)} (\ldots)}{p_k} + \sum_{\tau_i \in hp(k)} \frac{s_i}{p_i} \leq 1 \quad (6)$$

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Step 2: Partitioning tasks onto a multi-unit shared resource

An Integer linear programming based approach.

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Case Study

- Our shared-resource-centric partitioning framework is fully implemented on a GPU-enabled platform.
  - Ubuntu Liunx system with a 4-core Intel i7-4770 CPU
  - NVIDIA "Fermi" GTX 480 GPU (shared resource)
- Objectives: answer the following 3 questions
  - Whether it is practically feasible to resolve the problem from shared-resource-centric angle.
  - Whether solving this scheduling issue from the shared-resource-centric view is more effective than traditional CPU-centric view.
  - Whether our proposed schedulability tests can provide reliable predictability for given task systems.

Traditional CPU-centric Scheduling

Task	Para.	Period	GPU	CPU	Res. Tim
τ_1	12	6.40	0.80	1.00	N/A
τ_2	12	25.58	1.60	3.00	20.5
τ_3	2	12.79	1.00	2.00	N/A
τ_4	2	12.79	0.90	2.50	N/A
τ_5	4	19.19	1.80	2.00	17.70
τ_6	6	7.99	1.00	1.00	N/A
τ_7	6	8.99	0.80	2.00	N/A
τ_8	1	11.19	0.50	2.50	N/A

TABLE I: CPU-centric scheduling.

Traditional CPU-centric Scheduling

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τ_1	12	6.40	0.80	1.00	N/A
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τ_3	2	12.79	1.00	2.00	N/A
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τ_4	2	12.79	0.90	2.50	N/A
τ_5	4	19.19	1.80	2.00	17.70
τ_6	6	7.99	1.00	1.00	N/A
τ_7	6	8.99	0.80	2.00	N/A
τ_8	1	11.19	0.50	2.50	N/A

TABLE I: CPU-centric scheduling.

GPU-centric Scheduling (First-fit partitioning)

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1	1	12	6.40	0.80	1.00	5.8	Pass	Pass	Pass	12
τ_6		6	7.99	1.00	1.00	6.7	Pass	Pass	Pass	
τ_7		6	8.99	0.80	2.00	8.0	Pass	Pass	Pass	
τ_8	2	1	11.19	0.50	2.50	10.1	Pass	Pass	Pass	6
τ_4		2	12.79	0.90	2.50	11.2	Fail	Pass	Pass	
τ_3		2	12.79	1.00	2.00	11.9	Pass	Pass	Pass	
τ_5	3	4	19.19	1.80	2.00	14	Pass	Pass	Pass	12
τ_2		12	25.58	1.60	3.00	18.3	Pass	Fail	Pass	

TABLE II: Shared-resource-centric scheduling with first-fit partitioning using constant-time (CT) schedulability te

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.13	Pass	Pass	Pass	
τ_6	1	6	7.99	1.00	1.00	7.5	Pass	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.64	Pass	Fail	Pass	
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	
τ_4	2	2	12.79	0.90	2.50	12.2	Pass	Pass	Pass	2
τ_3		2	12.79	1.00	2.00	12.6	Fail	Pass	Pass	
τ_5	3	4	19.19	1.80	2.00	10.1	Pass	Pass	Pass	12
τ_2		12	25.58	1.60	3.00	11.8	Pass	Pass	Pass	

TABLE III: Shared-resource-centric scheduling with first-fit partitioning using TDA-based schedulability tests.

GPU-centric Scheduling (First-fit partitioning)

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1	1	12	6.40	0.80	1.00	5.8	Pass	Pass	Pass	12
τ_6		6	7.99	1.00	1.00	6.7	Pass	Pass	Pass	
τ_7		6	8.99	0.80	2.00	8.0	Pass	Pass	Pass	
τ_8	2	1	11.19	0.50	2.50	10.1	Pass	Pass	Pass	6
τ_4		2	12.79	0.90	2.50	11.2	Fail	Pass	Pass	
τ_3		2	12.79	1.00	2.00	11.9	Pass	Pass	Pass	
τ_5	3	4	19.19	1.80	2.00	14	Pass	Pass	Pass	12
τ_2		12	25.58	1.60	3.00	18.3	Pass	Fail	Pass	

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Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.13	Pass	Pass	Pass	
τ_6	1	6	7.99	1.00	1.00	7.5	Pass	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.64	Pass	Fail	Pass	
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	
τ_4	2	2	12.79	0.90	2.50	12.2	Pass	Pass	Pass	2
τ_3		2	12.79	1.00	2.00	12.6	Fail	Pass	Pass	
τ_5	3	4	19.19	1.80	2.00	10.1	Pass	Pass	Pass	12
τ_2		12	25.58	1.60	3.00	11.8	Pass	Pass	Pass	

TABLE III: Shared-resource-centric scheduling with first-fit partitioning using TDA-based schedulability tests.

GPU-centric Scheduling (First-fit partitioning)

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1	1	12	6.40	0.80	1.00	5.8	Pass	Pass	Pass	12
τ_6		6	7.99	1.00	1.00	6.7	Pass	Pass	Pass	
τ_7		6	8.99	0.80	2.00	8.0	Pass	Pass	Pass	
τ_8	2	1	11.19	0.50	2.50	10.1	Pass	Pass	Pass	6
τ_4		2	12.79	0.90	2.50	11.2	Fail	Pass	Pass	
τ_3		2	12.79	1.00	2.00	11.9	Pass	Pass	Pass	
τ_5	3	4	19.19	1.80	2.00	14	Pass	Pass	Pass	12
τ_2		12	25.58	1.60	3.00	18.3	Pass	Fail	Pass	

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Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.13	Pass	Pass	Pass	
τ_6	1	6	7.99	1.00	1.00	7.5	Pass	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.64	Pass	Fail	Pass	
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	
τ_4	2	2	12.79	0.90	2.50	12.2	Pass	Pass	Pass	2
τ_3		2	12.79	1.00	2.00	12.6	Fail	Pass	Pass	
τ_5	3	4	19.19	1.80	2.00	10.1	Pass	Pass	Pass	12
τ_2		12	25.58	1.60	3.00	11.8	Pass	Pass	Pass	

TABLE III: Shared-resource-centric scheduling with first-fit partitioning using TDA-based schedulability tests.

GPU-centric Scheduling (First-fit partitioning)

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1	1	12	6.40	0.80	1.00	5.8	Pass	Pass	Pass	12
τ_6		6	7.99	1.00	1.00	6.7	Pass	Pass	Pass	
τ_7		6	8.99	0.80	2.00	8.0	Pass	Pass	Pass	
τ_8	2	1	11.19	0.50	2.50	10.1	Pass	Pass	Pass	6
τ_4		2	12.79	0.90	2.50	11.2	Fail	Pass	Pass	
τ_3		2	12.79	1.00	2.00	11.9	Pass	Pass	Pass	
τ_5	3	4	19.19	1.80	2.00	14	Pass	Pass	Pass	12
τ_2		12	25.58	1.60	3.00	18.3	Pass	Fail	Pass	

TABLE II: Shared-resource-centric scheduling with first-fit partitioning using constant-time (CT) schedulability te

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.13	Pass	Pass	Pass	
τ_6	1	6	7.99	1.00	1.00	7.5	Pass	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.64	Pass	Fail	Pass	
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	
τ_4	2	2	12.79	0.90	2.50	12.2	Pass	Pass	Pass	2
τ_3		2	12.79	1.00	2.00	12.6	Fail	Pass	Pass	
τ_5	3	4	19.19	1.80	2.00	10.1	Pass	Pass	Pass	12
τ_2		12	25.58	1.60	3.00	11.8	Pass	Pass	Pass	

TABLE III: Shared-resource-centric scheduling with first-fit partitioning using TDA-based schedulability tests.

GPU-centric Scheduling (First-fit partitioning)

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Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1	1	12	6.40	0.80	1.00	5.8	Pass	Pass	Pass	12
τ_6		6	7.99	1.00	1.00	6.7	Pass	Pass	Pass	
τ_7		6	8.99	0.80	2.00	8.0	Pass	Pass	Pass	
τ_8	2	1	11.19	0.50	2.50	10.1	Pass	Pass	Pass	6
τ_4		2	12.79	0.90	2.50	11.2	Fail	Pass	Pass	
τ_3		2	12.79	1.00	2.00	11.9	Pass	Pass	Pass	
τ_5	3	4	19.19	1.80	2.00	14	Pass	Pass	Pass	12
τ_2		12	25.58	1.60	3.00	18.3	Pass	Fail	Pass	

TABLE II: Shared-resource-centric scheduling with first-fit partitioning using constant-time (CT) schedulability te

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Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.13	Pass	Pass	Pass	
τ_6	1	6	7.99	1.00	1.00	7.5	Pass	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.64	Pass	Fail	Pass	
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	
τ_4	2	2	12.79	0.90	2.50	12.2	Pass	Pass	Pass	2
τ_3		2	12.79	1.00	2.00	12.6	Fail	Pass	Pass	
τ_5	3	4	19.19	1.80	2.00	10.1	Pass	Pass	Pass	12
τ_2		12	25.58	1.60	3.00	11.8	Pass	Pass	Pass	

TABLE III: Shared-resource-centric scheduling with first-fit partitioning using TDA-based schedulability tests.

GPU-centric Scheduling (ILP-based partitioning)

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.03	Pass	Pass	Pass	
τ_2	1	12	25.58	1.60	3.00	17.71	Fail	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.32	Pass	Pass	Pass	
τ_6	2	6	7.99	1.00	1.00	7.45	Pass	Pass	Pass	6
τ_5		4	19.19	1.80	2.00	16.6	Pass	Pass	Pass	
τ_3		2	12.79	2.00	2.00	12.6	Pass	Pass	Pass	
τ_4	3	2	12.79	0.90	2.50	12.2	Fail	Pass	Pass	2
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	

TABLE IV: Shared-resource-centric scheduling with Algorithm 2 using CT schedulability tests.

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.03	Pass	Pass	Pass	
τ_2	1	12	25.58	1.60	3.00	17.71	Pass	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.32	Pass	Pass	Pass	
τ_6	2	6	7.99	1.00	1.00	7.45	Pass	Pass	Pass	6
τ_5		4	19.19	1.80	2.00	16.6	Pass	Pass	Pass	
τ_3		2	12.79	2.00	2.00	12.6	Pass	Pass	Pass	
τ_4	3	2	12.79	0.90	2.50	12.2	Fail	Pass	Pass	2
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	

TABLE V: Shared-resource-centric scheduling with Algorithm 2 using TDA-based schedulability tests.

GPU-centric Scheduling (ILP-based partitioning)

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.03	Pass	Pass	Pass	
τ_2	1	12	25.58	1.60	3.00	17.71	Fail	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.32	Pass	Pass	Pass	
τ_6	2	6	7.99	1.00	1.00	7.45	Pass	Pass	Pass	6
τ_5		4	19.19	1.80	2.00	16.6	Pass	Pass	Pass	
τ_3		2	12.79	2.00	2.00	12.6	Pass	Pass	Pass	
τ_4	3	2	12.79	0.90	2.50	12.2	Fail	Pass	Pass	2
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	

TABLE IV: Shared-resource-centric scheduling with Algorithm 2 using CT schedulability tests.

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.03	Pass	Pass	Pass	
τ_2	1	12	25.58	1.60	3.00	17.71	Pass	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.32	Pass	Pass	Pass	
τ_6	2	6	7.99	1.00	1.00	7.45	Pass	Pass	Pass	6
τ_5		4	19.19	1.80	2.00	16.6	Pass	Pass	Pass	
τ_3		2	12.79	2.00	2.00	12.6	Pass	Pass	Pass	
τ_4	3	2	12.79	0.90	2.50	12.2	Fail	Pass	Pass	2
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	

TABLE V: Shared-resource-centric scheduling with Algorithm 2 using TDA-based schedulability tests.

GPU-centric Scheduling (ILP-based partitioning)

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.03	Pass	Pass	Pass	
τ_2	1	12	25.58	1.60	3.00	17.71	Fail	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.32	Pass	Pass	Pass	
τ_6	2	6	7.99	1.00	1.00	7.45	Pass	Pass	Pass	6
τ_5		4	19.19	1.80	2.00	16.6	Pass	Pass	Pass	
τ_3		2	12.79	2.00	2.00	12.6	Pass	Pass	Pass	
τ_4	3	2	12.79	0.90	2.50	12.2	Fail	Pass	Pass	2
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	

TABLE IV: Shared-resource-centric scheduling with Algorithm 2 using CT schedulability tests.

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.03	Pass	Pass	Pass	
τ_2	1	12	25.58	1.60	3.00	17.71	Pass	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.32	Pass	Pass	Pass	
τ_6	2	6	7.99	1.00	1.00	7.45	Pass	Pass	Pass	6
τ_5		4	19.19	1.80	2.00	16.6	Pass	Pass	Pass	
τ_3		2	12.79	2.00	2.00	12.6	Pass	Pass	Pass	
τ_4	3	2	12.79	0.90	2.50	12.2	Fail	Pass	Pass	2
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	

TABLE V: Shared-resource-centric scheduling with Algorithm 2 using TDA-based schedulability tests.

GPU-centric Scheduling (ILP-based partitioning)

Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.03	Pass	Pass	Pass	
τ_2	1	12	25.58	1.60	3.00	17.71	Fail	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.32	Pass	Pass	Pass	
τ_6	2	6	7.99	1.00	1.00	7.45	Pass	Pass	Pass	6
τ_5		4	19.19	1.80	2.00	16.6	Pass	Pass	Pass	
τ_3		2	12.79	2.00	2.00	12.6	Pass	Pass	Pass	
τ_4	3	2	12.79	0.90	2.50	12.2	Fail	Pass	Pass	2
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	

TABLE IV: Shared-resource-centric scheduling with Algorithm 2 using CT schedulability tests.

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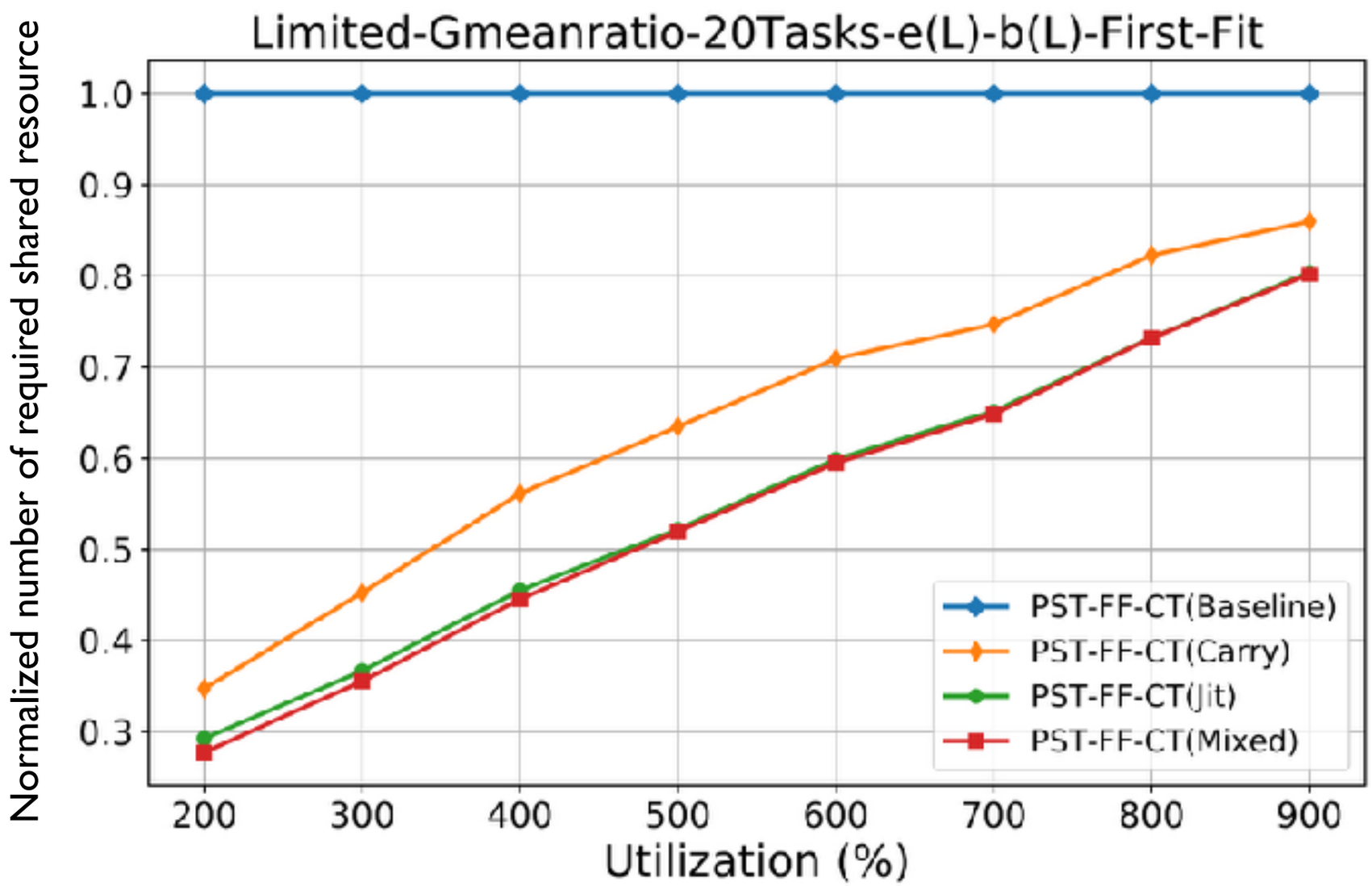
Task	Part.	Para.	Period	GPU	CPU	Res. Time	Carry	Jitter	Mixed	Warps
τ_1		12	6.40	0.80	1.00	6.03	Pass	Pass	Pass	
τ_2	1	12	25.58	1.60	3.00	17.71	Pass	Pass	Pass	12
τ_7		6	8.99	0.80	2.00	8.32	Pass	Pass	Pass	
τ_6	2	6	7.99	1.00	1.00	7.45	Pass	Pass	Pass	6
τ_5		4	19.19	1.80	2.00	16.6	Pass	Pass	Pass	
τ_3		2	12.79	2.00	2.00	12.6	Pass	Pass	Pass	
τ_4	3	2	12.79	0.90	2.50	12.2	Fail	Pass	Pass	2
τ_8		1	11.19	0.50	2.50	10.5	Pass	Pass	Pass	

TABLE V: Shared-resource-centric scheduling with Algorithm 2 using TDA-based schedulability tests.

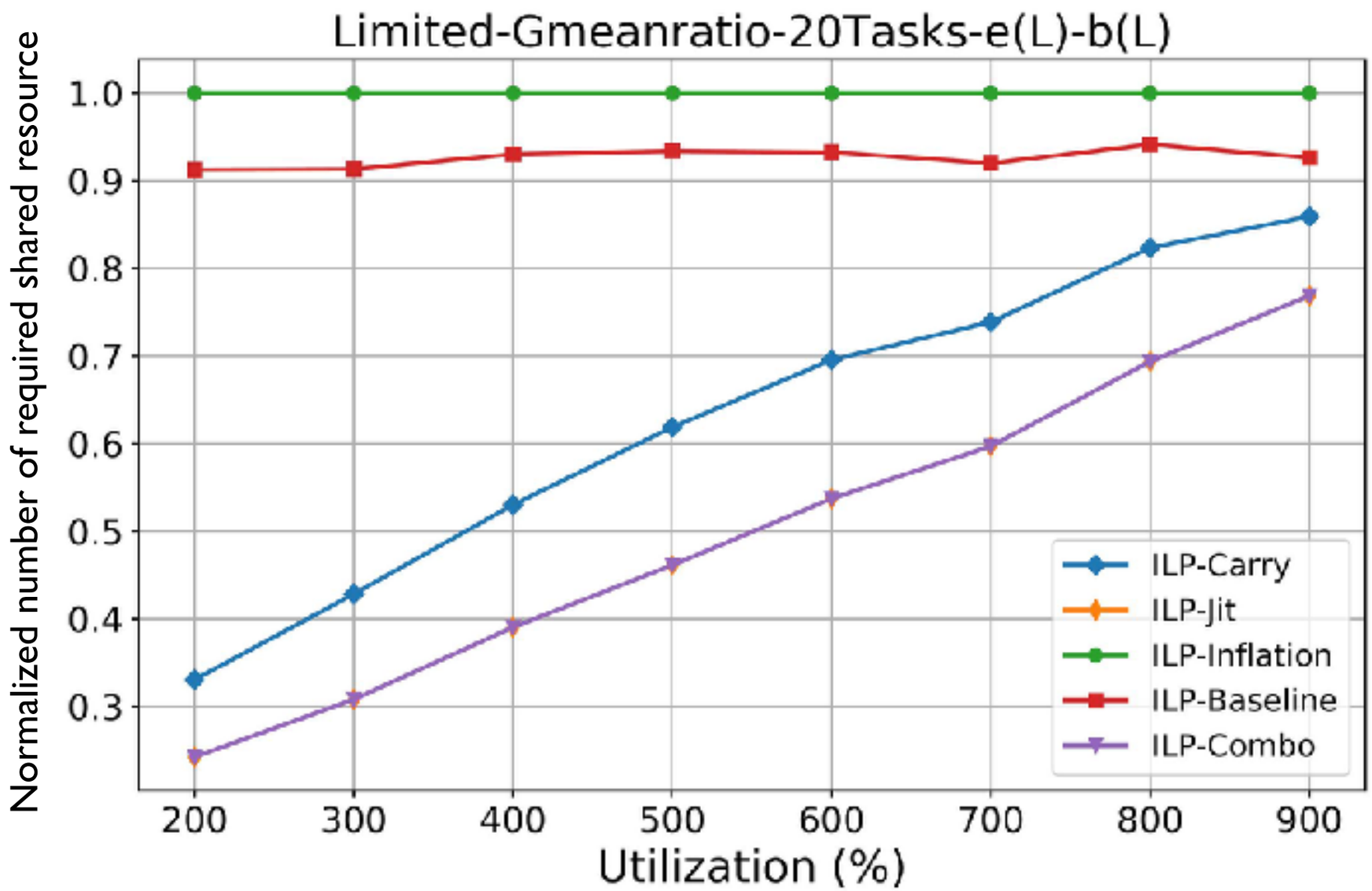
Simulation evaluation

- The UUnifast-Discard method was applied to generate task sets.
- Evaluated methods
  - Comparison with first-fit partition using TDA-based tests.
  - Comparison with first-fit partition using constant time tests.
  - Comparison among different ILP-based approaches.
  - Representative Approaches Comparison.

First-Fit partition using TDA-based tests(CT tests).



ILP-based approaches and representative approaches comparison



Conclusion

- Develop a HRT multi-phase task scheduling framework
  - Resolve this problem from a counter-intuitive shared-resource-centric perspective.
  - Develop a rather comprehensive set of suspending task partitioning algorithms that minimize the required size of the shared resource.
  - A GPU-based prototype case study and extensive simulation-based experiments have been conducted.

Thank you!